Global fits of nuclear PDFs

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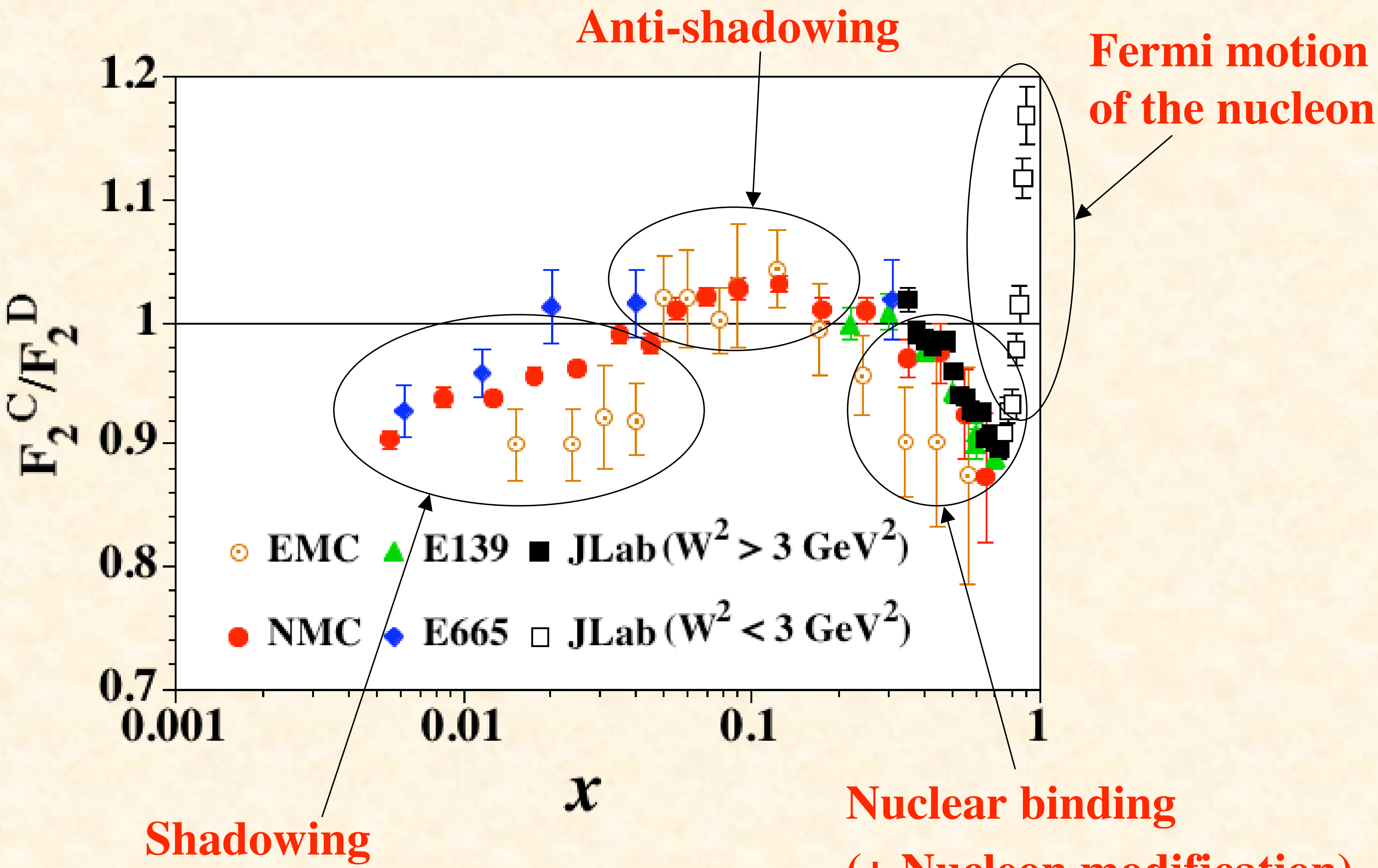
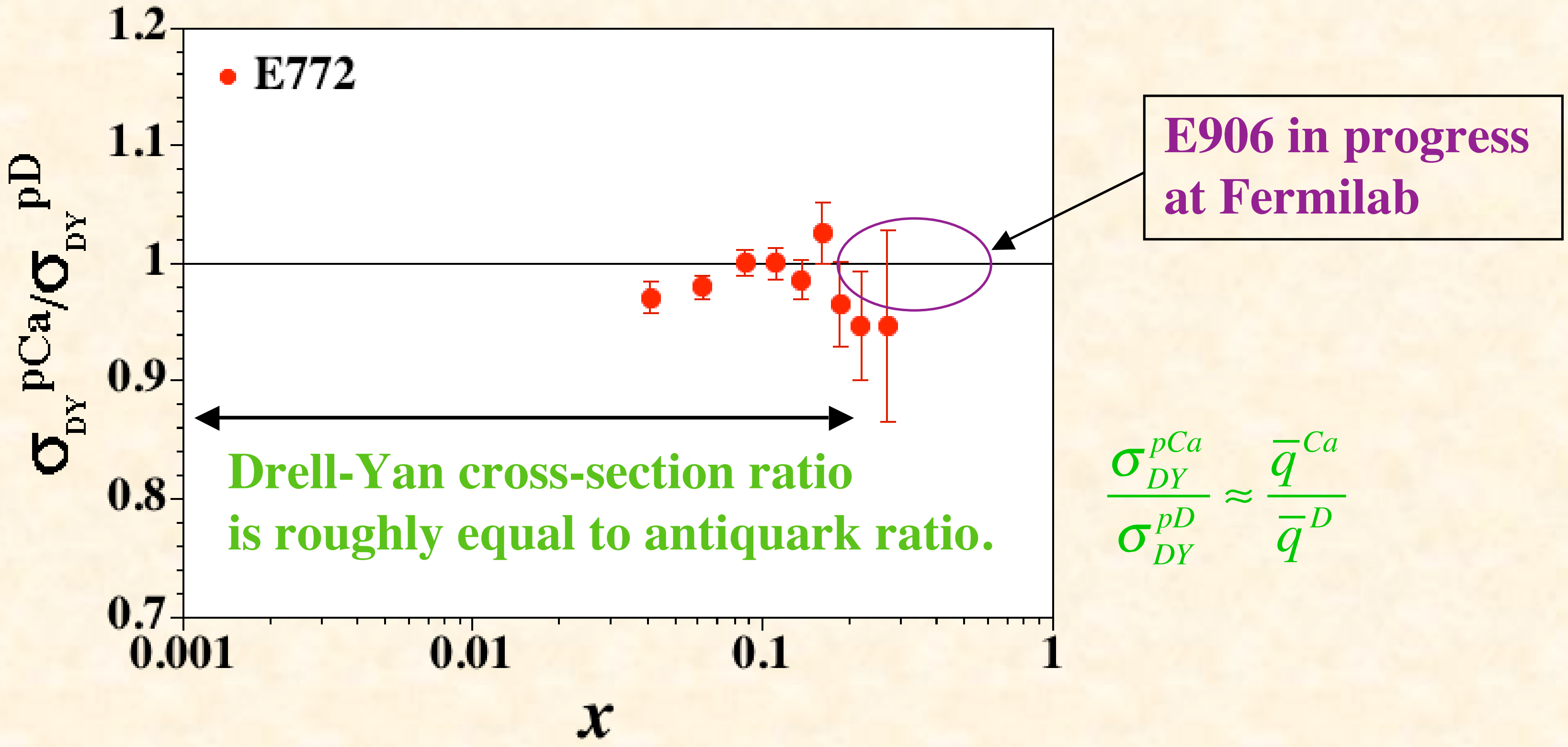
- Analysis on **tensor-polarized PDFs** in the deuteron

Nuclear modifications of structure function F_2



Drell-Yan and Antiquark Distributions

$$p + A \rightarrow \mu^+ \mu^- + X$$

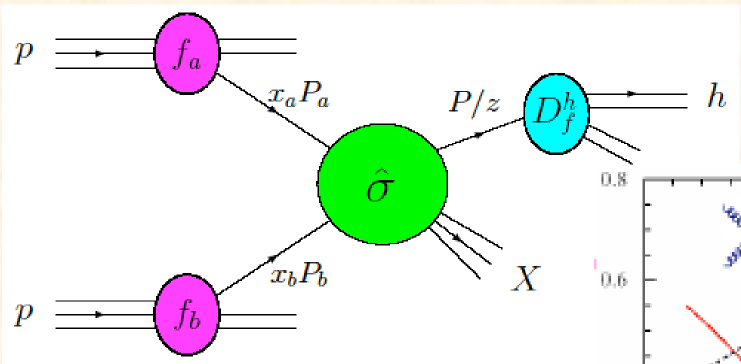


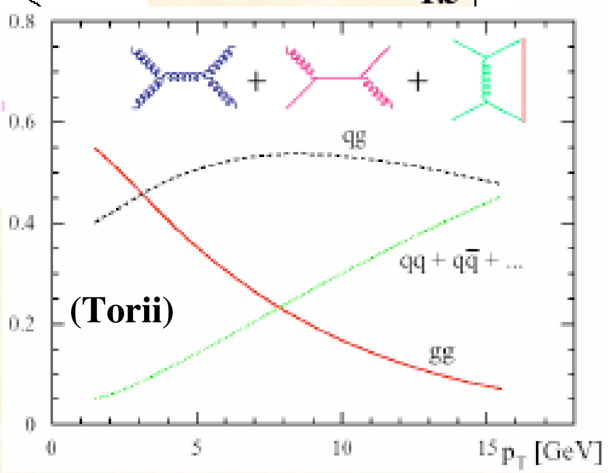
$$\frac{\sigma_{DY}^{pCa}}{\sigma_{DY}^{pD}} \approx \frac{\bar{q}^{Ca}}{\bar{q}^{D}}$$

The Fermilab E772 Drell-Yan data suggested that nuclear modification of antiquark distributions should be small in the region, x≈0.1.

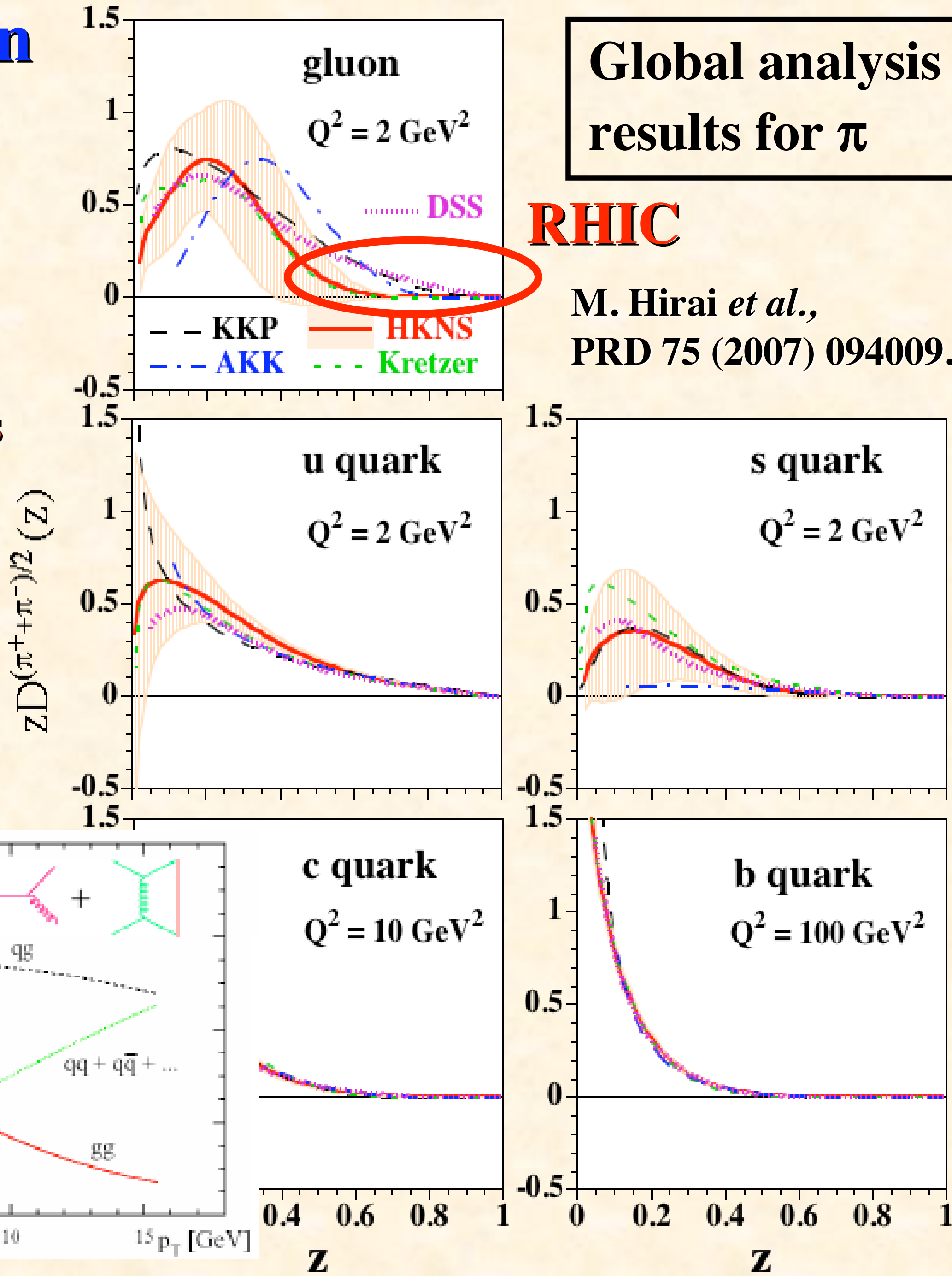
Uncertainties of fragmentation functions "in including hadron -production data in the global analysis."

- Gluon and light-quark fragmentation functions have large uncertainties.
- Large differences between the functions of various analysis groups.
- Gluon function at large-z is important for hadron-productions at RHIC.





Global analysis results for π



M. Hirai *et al.*, PRD 75 (2007) 094009.

Determination of Nuclear Parton Distribution Functions

(1) M. Hirai, S. Kumano, M. Miyama, Phys. Rev. D64 (2001) 034003.

(2) M. Hirai, S. Kumano, T.-H. Nagai, Phys. Rev. C70 (2004) 044905.

(3) M. Hirai, S. Kumano, T.-H. Nagai, Phys. Rev. C76 (2007) 065207.

Research in progress …

Experimental data: total number = 1241

(1) F_2^A / F_2^D 896 data

- **NMC:** p, He, Li, C, Ca
- **SLAC:** He, Be, C, Al, Ca, Fe, Ag, Au
- **EMC:** C, Ca, Cu, Sn
- **E665:** C, Ca, Xe, Pb
- **BCDMS:** N, Fe
- **HERMES:** N, Kr

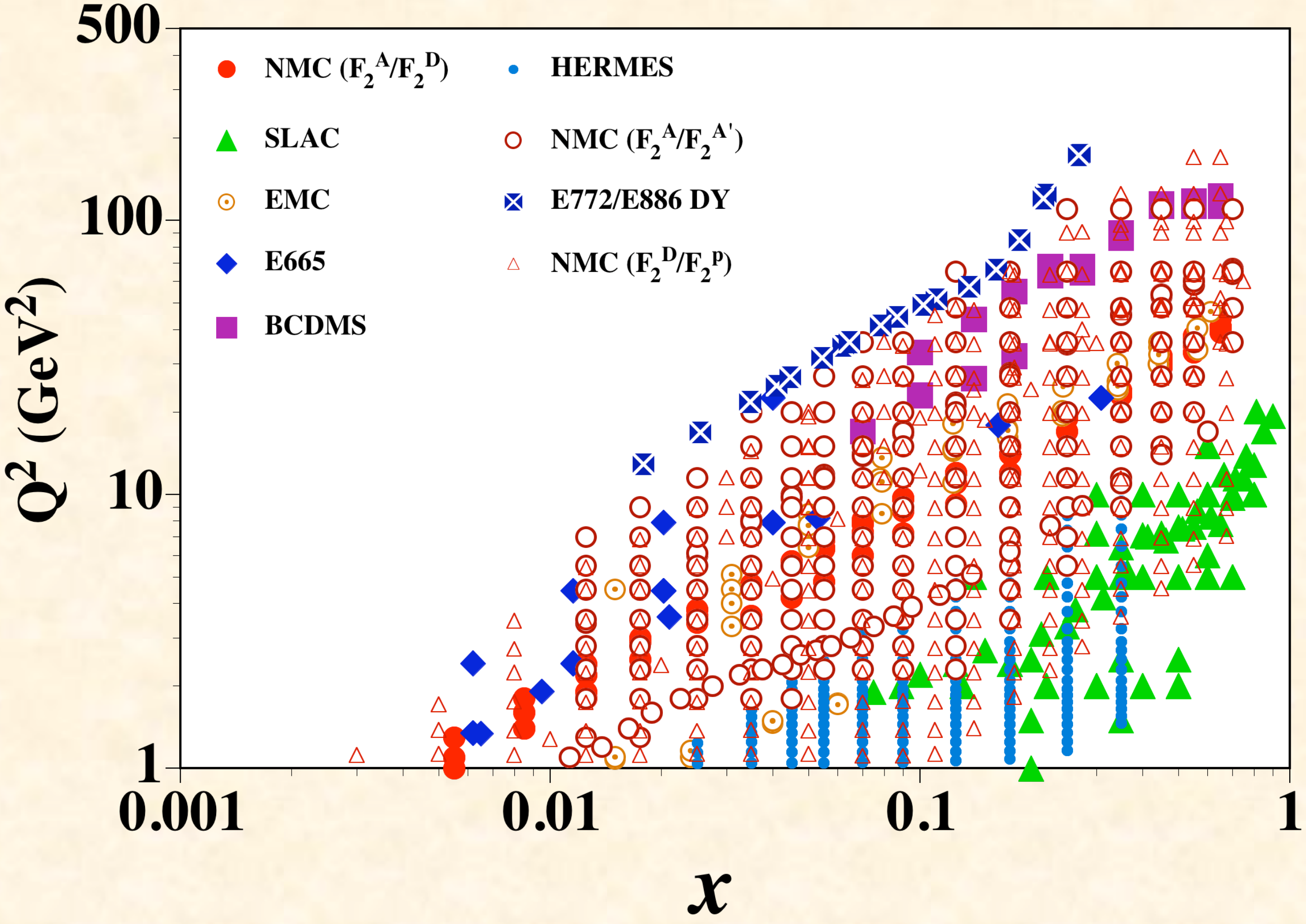
(2) $F_2^A / F_2^{A'}$ 293 data

- **NMC:** Be / C, Al / C, Ca / C, Fe / C, Sn / C, Pb / C, C / Li, Ca / Li

(3) $\sigma_{DY}^A / \sigma_{DY}^{A'}$ 52 data

- **E772:** C / D, Ca / D, Fe / D, W / D
- **E866:** Fe / Be, W / Be

+ JLab data



Functional form

Nuclear PDFs "per nucleon"

If there were no nuclear modification

$$Au^A(x) = Zu^p(x) + Nu^n(x), \quad Ad^A(x) = Zd^p(x) + Nd^n(x) \qquad p = \text{proton}, \quad n = \text{neutron}$$

Isospin symmetry: $u^n = d^p \equiv d, \qquad d^n = u^p \equiv u$

$$\rightarrow \quad u^A(x) = \frac{Zu(x) + Nd(x)}{A}, \qquad d^A(x) = \frac{Zd(x) + Nu(x)}{A}$$

Take account of nuclear effects by $w_i(x, A)$

$$u_v^A(x) = w_{u_v}(x,A)\frac{Zu_v(x) + Nd_v(x)}{A}, \quad d_v^A(x) = w_{d_v}(x,A)\frac{Zd_v(x) + Nu_v(x)}{A}$$

$$\bar{u}^A(x) = w_{\bar{q}}(x,A)\frac{Z\bar{u}(x) + N\bar{d}(x)}{A}, \qquad \bar{d}^A(x) = w_{\bar{q}}(x,A)\frac{Z\bar{d}(x) + N\bar{u}(x)}{A}$$

$$\bar{s}^A(x) = w_{\bar{q}}(x,A)\bar{s}(x)$$

$$g^A(x) = w_g(x,A)g(x)$$

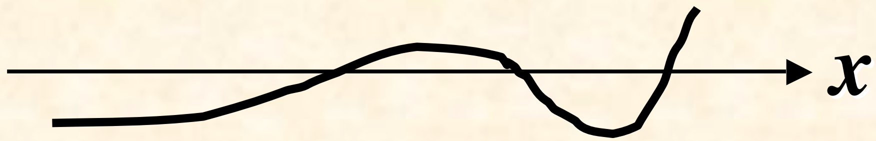
at Q^2=1 GeV^2 ($\equiv Q_0^2$)

Functional form of $w_i(x, A)$

$$f_i^A(x,Q_0^2) = w_i(x,A) f_i(x,Q_0^2) \qquad i = u_v,\ d_v,\ \bar{u},\ \bar{d},\ \bar{s},\ g$$

$$w_i(x,A) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^\beta}$$

Note: The region $x > 1$ cannot be described by this parametrization.



A simple function = cubic polynomial

Three constraints

Nuclear charge: $Z = A\int dx \left[\frac{2}{3}\left(u^A - \bar{u}^A\right) - \frac{1}{3}\left(d^A - \bar{d}^A\right) - \frac{1}{3}\left(s^A - \bar{s}^A\right)\right] = A\int dx \left[\frac{2}{3}u_v^A - \frac{1}{3}d_v^A\right]$

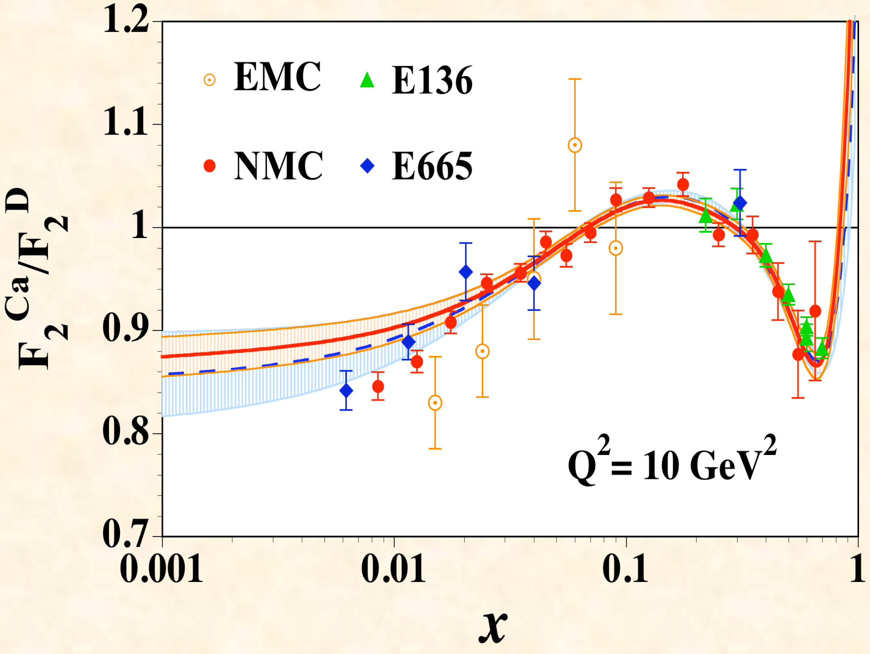
Baryon number: $A = A\int dx \left[\frac{1}{3}\left(u^A - \bar{u}^A\right) + \frac{1}{3}\left(d^A - \bar{d}^A\right) + \frac{1}{3}\left(s^A - \bar{s}^A\right)\right] = A\int dx \left[\frac{1}{3}u_v^A + \frac{1}{3}d_v^A\right]$

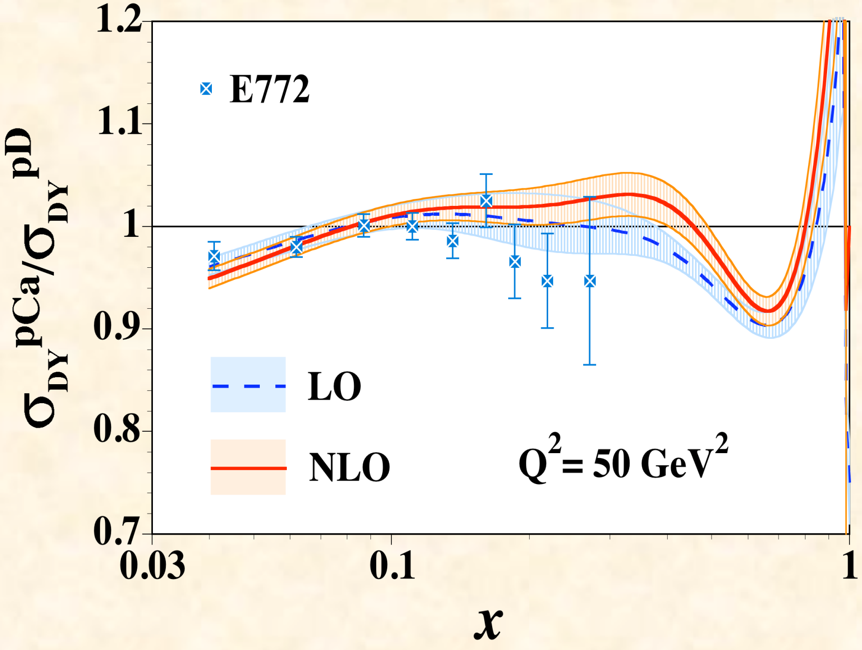
Momentum:

$$A = A\int dx \left[u^A + \bar{u}^A + d^A + \bar{d}^A + s^A + \bar{s}^A + g\right]$$
$$= A\int dx \left[u_v^A + d_v^A + 2\left(\bar{u}^A + \bar{d}^A + \bar{s}^A\right) + g\right]$$

Comparison with F_2^{Ca}/F_2^{D} & $\sigma_{DY}^{pCa}/\sigma_{DY}^{pD}$ data

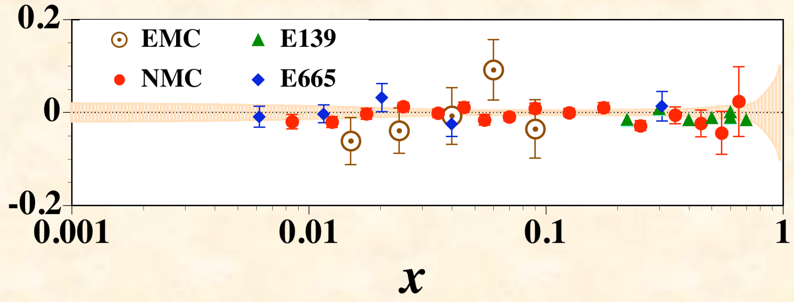
LO analysis **NLO analysis**

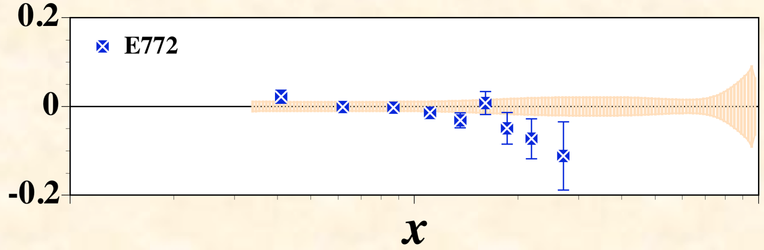




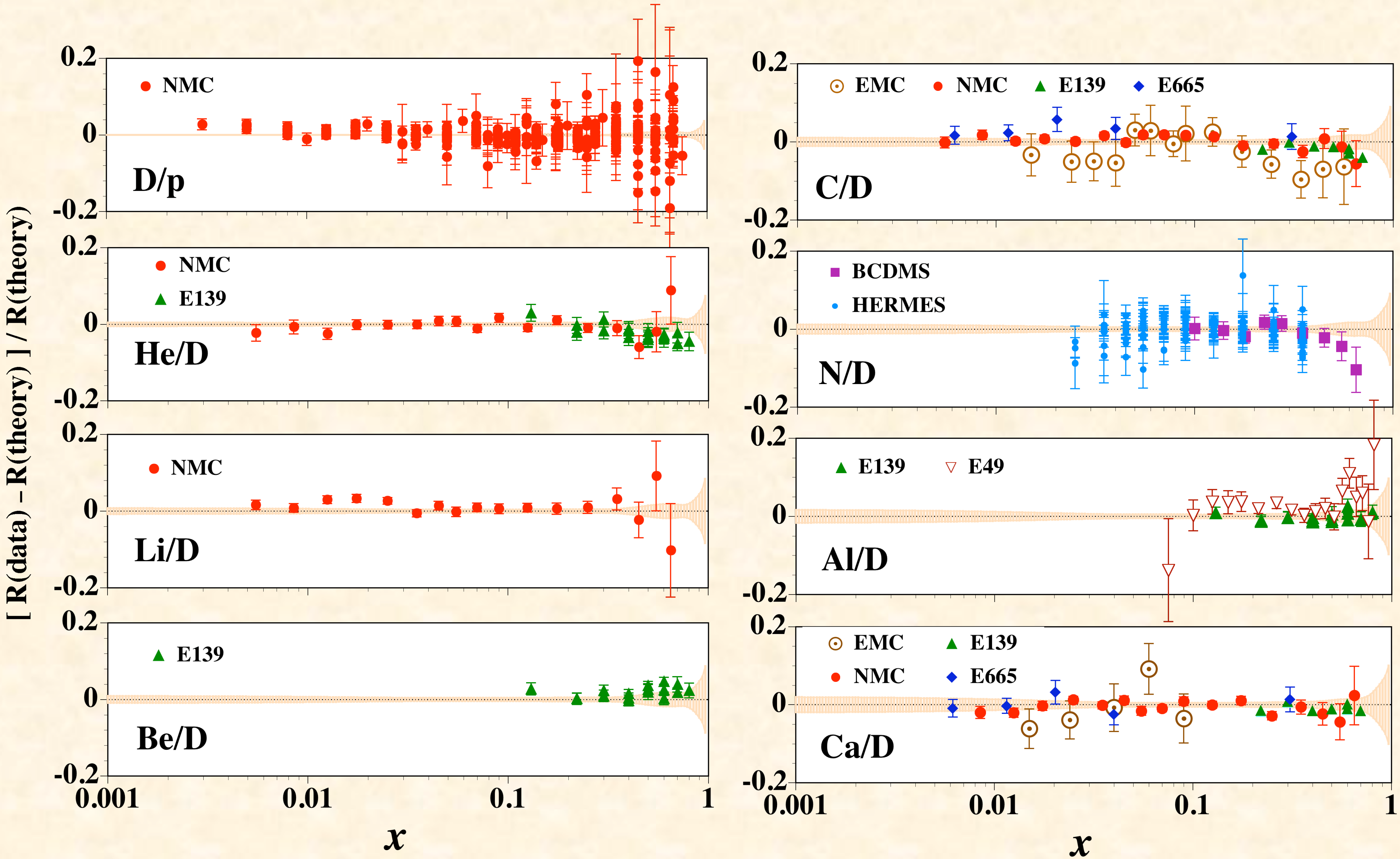
$(R^{exp}-R^{theo})/R^{theo}$ at the same Q^2 points

$R= F_2^{Ca}/F_2^{D},\ \sigma_{DY}^{pCa}/\sigma_{DY}^{pD}$

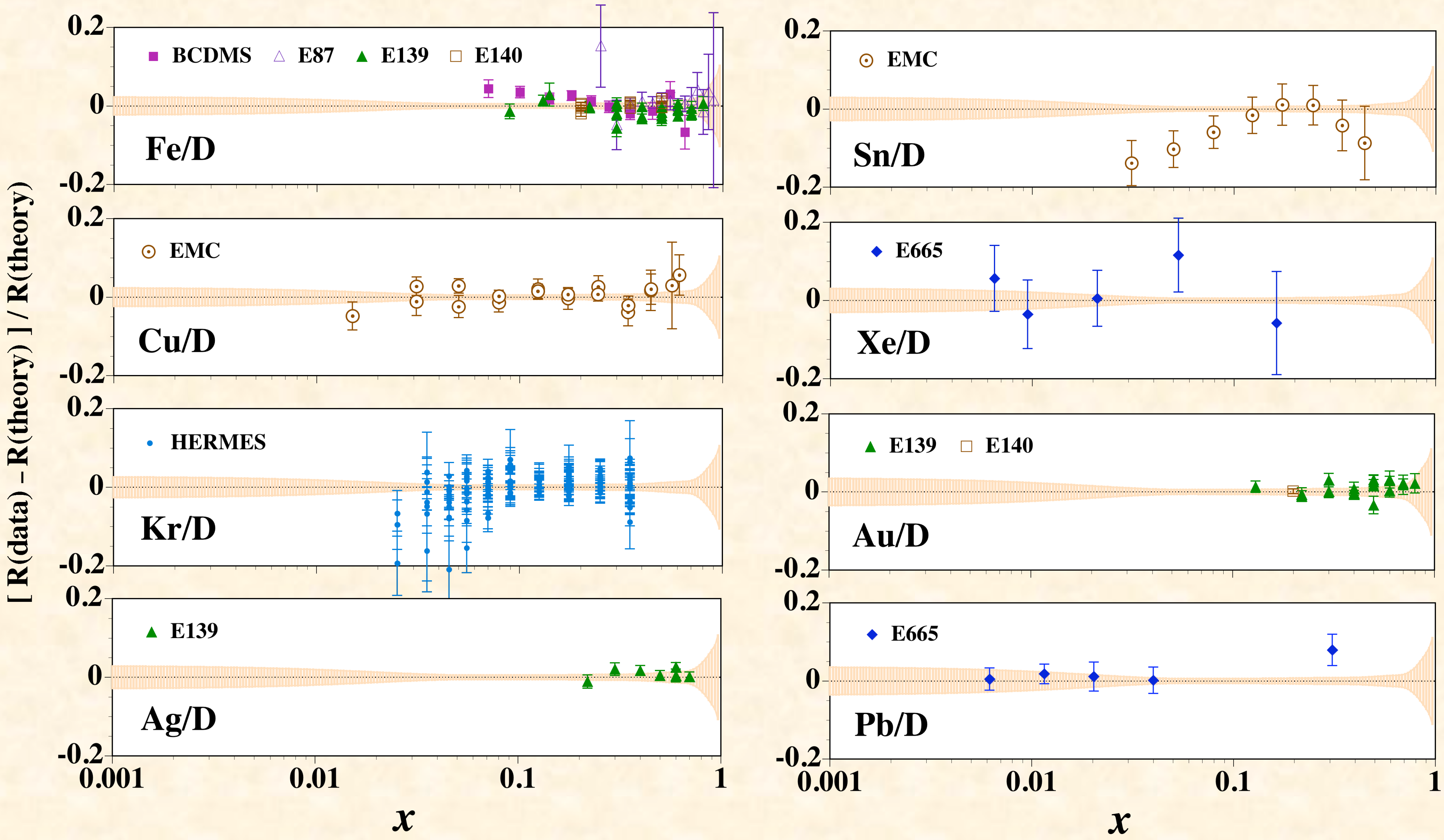




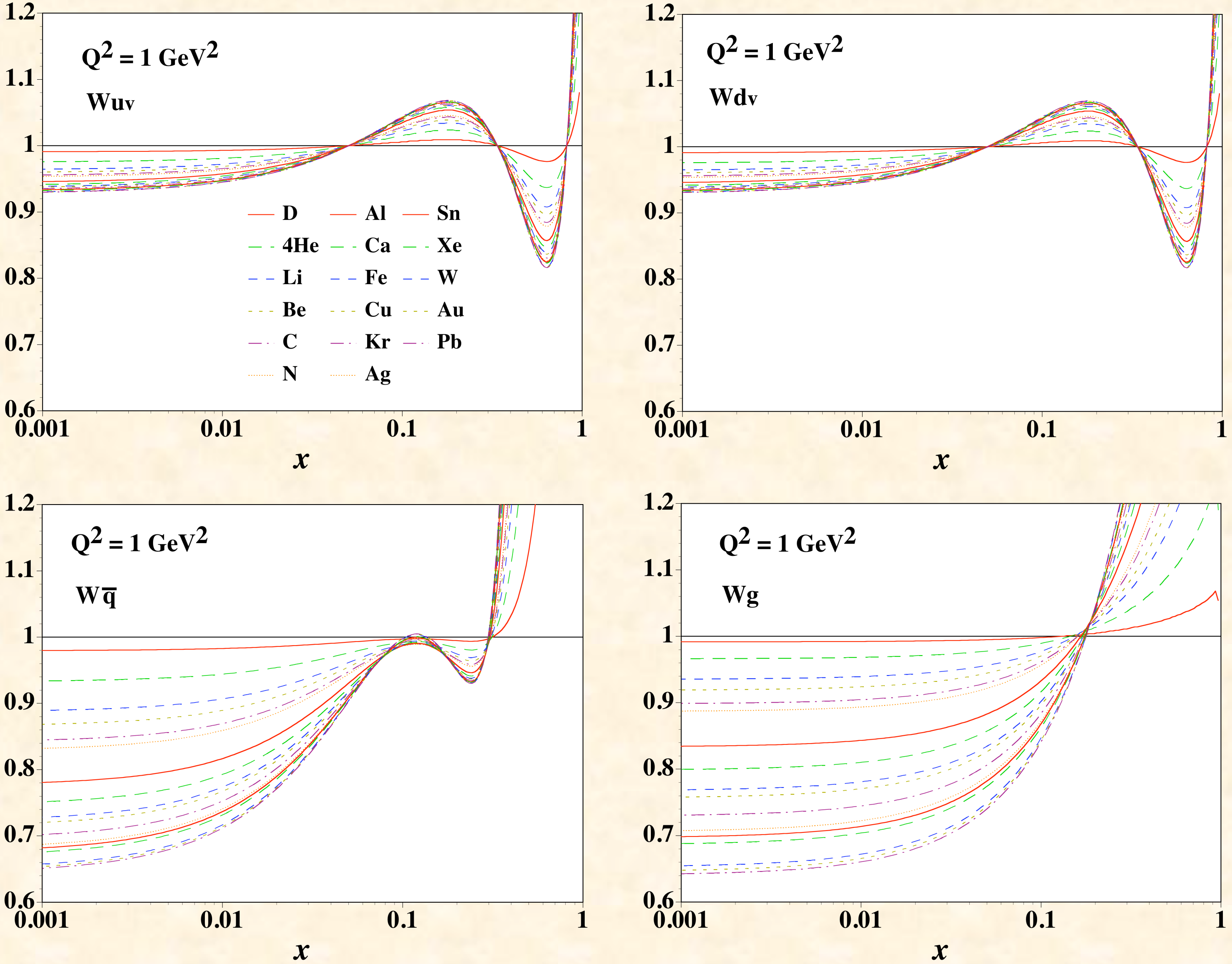
Comparison with F_2^A/F_2^D data: Light nuclei



Comparison with F_2^A/F_2^D data: Heavy nuclei

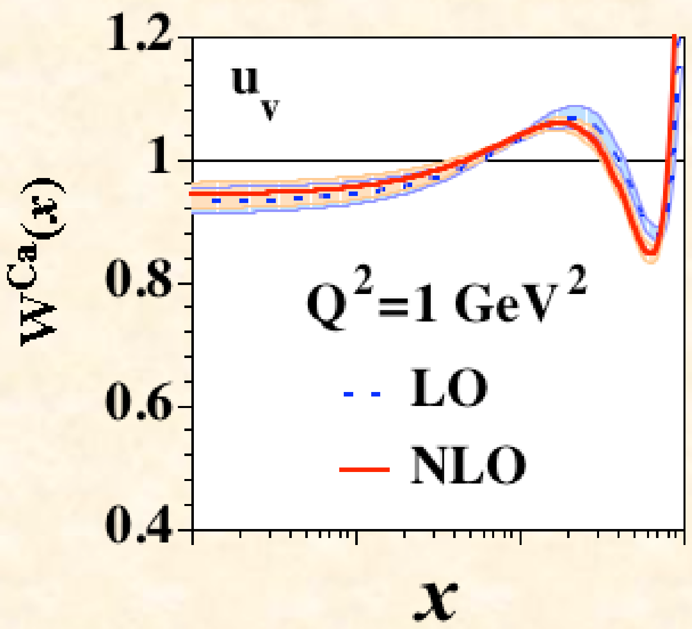


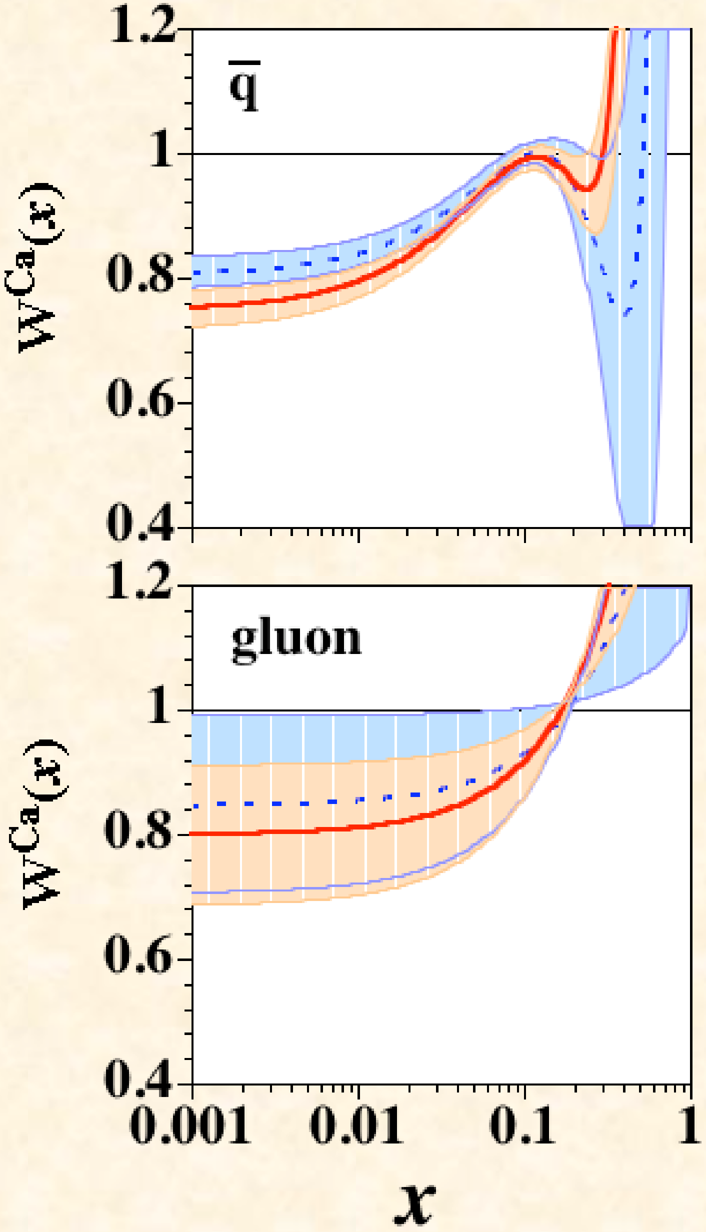
Nuclear PDFs



Nuclear PDFs and their uncertainties

- Some NLO improvements, but not significant ones.
- Impossible to determine gluon modifications.
- Antiquark distributions are not determined at large *x*.



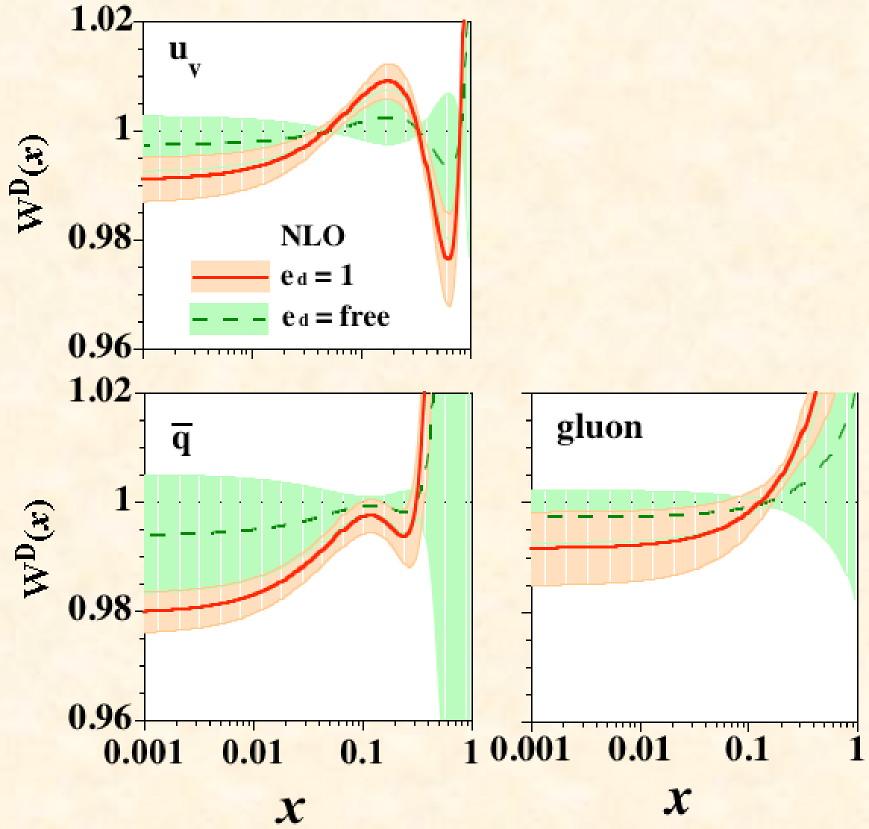


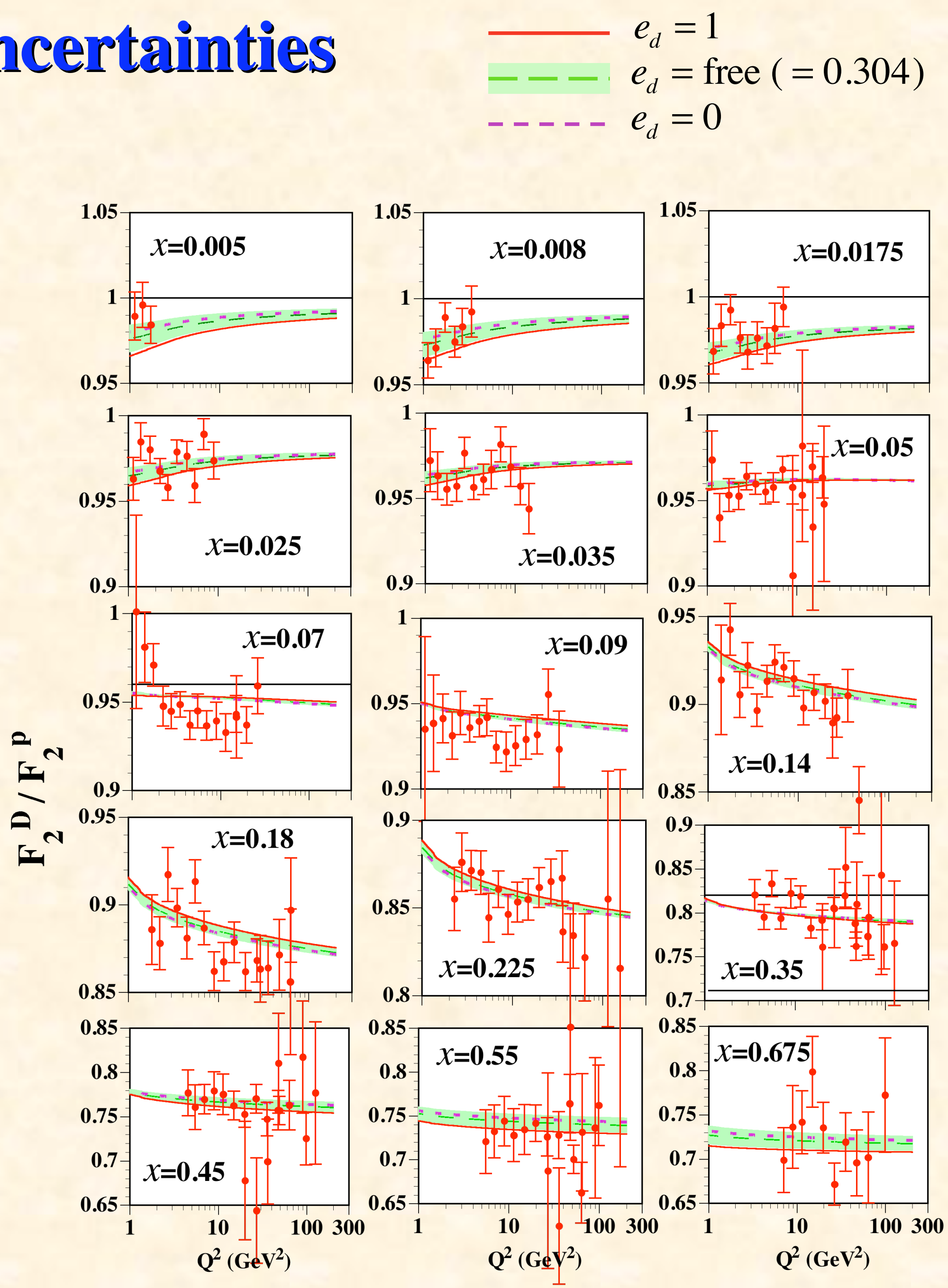
PDFs in deuteron and their uncertainties

$$w_i(x,A) = 1 + e_d \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^\beta}$$

e_d = free ≠ 0 for D, $e_d = 0$ for other nuclei

- **Deuteron effects are partially included in the nucleonic PDFs.**
- **At most, 0.5–2% modifications.**





Recent global analyses on nuclear PDFs

It is likely that I miss some papers!

- **EPS09**
  - K. J. Eskola, H. Paukkunen, and C. A. Salgado, JHEP 04 (2009) 065.
  - Charged-lepton DIS, DY, π^0 production in *dAu*.
- **SYKMOO08 (09)**
  - I. Schienbein, J. Y. Yu, C. Keppel, J. G. Morfin, F. I. Olness, and J. F. Owens, Phys. Rev. D 77 (2008) 044013; D80 (2009) 094004.
  - Neutrino DIS (only NuTeV data).
- **HKN07**
  - M. Hirai, S. Kumano, and T. -H. Nagai, Phys. Rev. C 76 (2007) 065207.
  - Charged-lepton DIS, DY.
- **DS04**
  - D. de Florian and R. Sassot, Phys. Rev. D 69 (2004) 074028.
  - Charged-lepton DIS, DY.

See also L. Frankfurt, V. Guzey, and M. Strikman, Phys. Rev. D 71 (2005) 054001;
Phys. Lett. B687 (2010) 167.
S. A. Kulagin and R. Petti, Phys. Rev. D 76 (2007) 094023.

Comparison of nuclear PDFs

Different analysis results are consistent with each other because they are roughly within uncertainty bands.

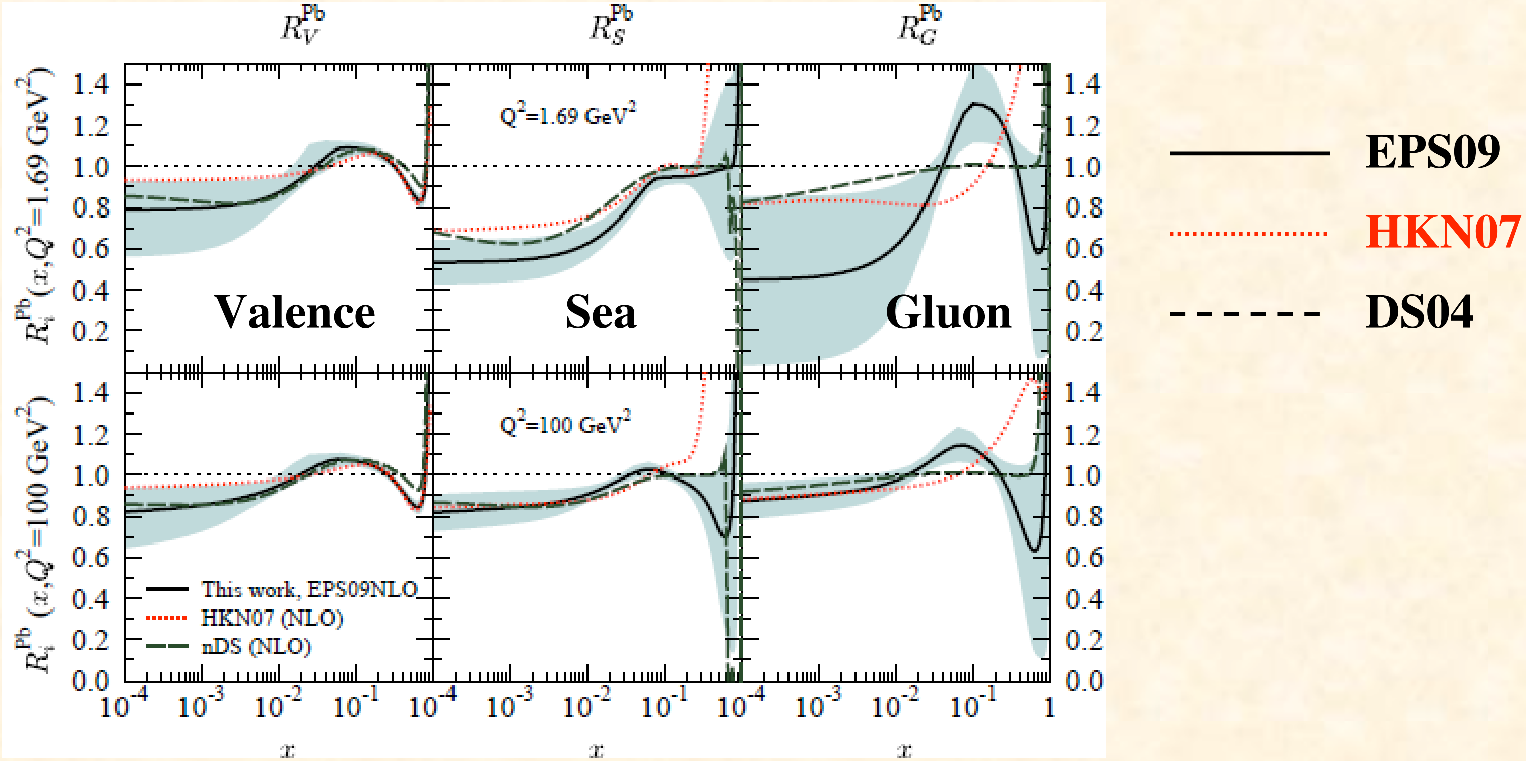
Valence quark: **Well determined except at small *x*.**

Antiquark: **Determined at small *x*, Large uncertainties at medium and large *x*.**

Gluon: **Large uncertainties in the whole-*x* region.**

$Q^2 = 1.69\ \mathrm{GeV}^2$

$Q^2 = 100\ \mathrm{GeV}^2$



EPS09 (K. J. Eskola *et al.*), JHEP 04 (2009) 065

Summary on nuclear-PDF determination in NLO

LO and NLO analysis for the nuclear PDFs and their uncertainties.

Valence quark: **well determined**

Antiquark: **determined at small x, large uncertainties at medium and large x.**

Gluon: **large uncertainties in the whole-x region.**

- **Better determination of $G^A(x)$ is usually expected in NLO.**

→ **However, the NLO improvement is not very clear due to inaccurate measurement of Q^2 dependence.**

→ **The gluon modifications are not well determined even in NLO.**

Deuteron modifications

- **At most 0.5%~2%; however, be careful that deuteron effects could be contained in the PDFs of the nucleon.**

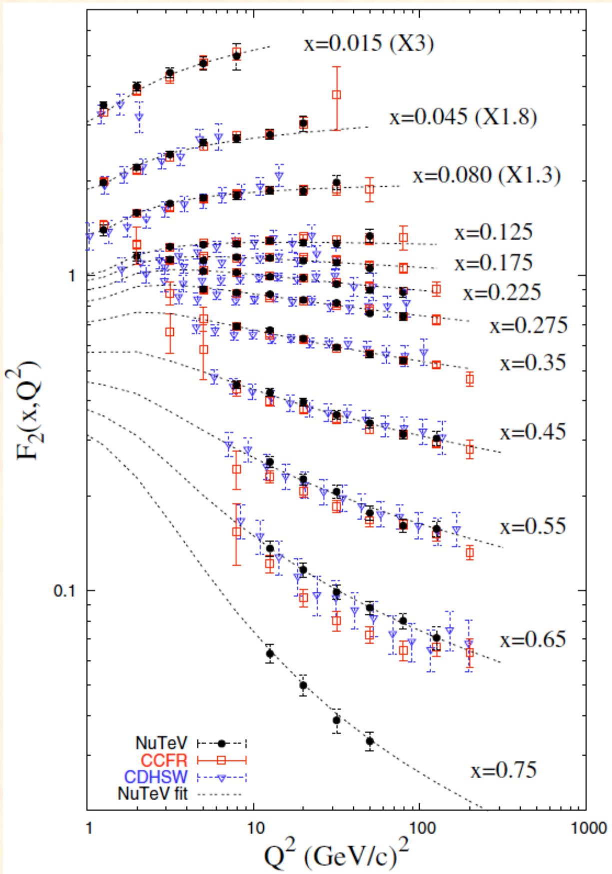
NPDF codes at http://research.kek.jp/people/kumanos/nuclp.html.

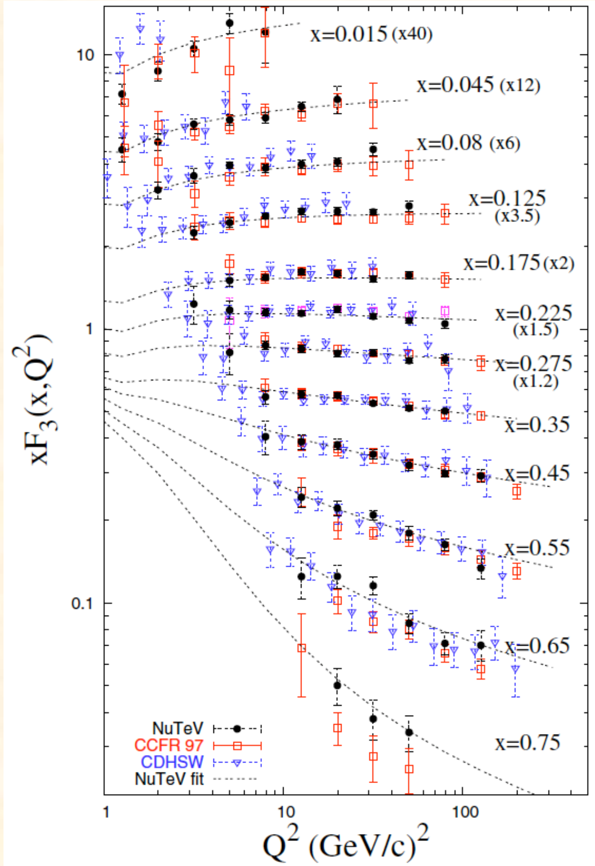
Recent neutrino DIS experiments

Experiment	Target	ν energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

M. Tzanov *et al.* (NuTeV), PRD74 (2006) 012008.

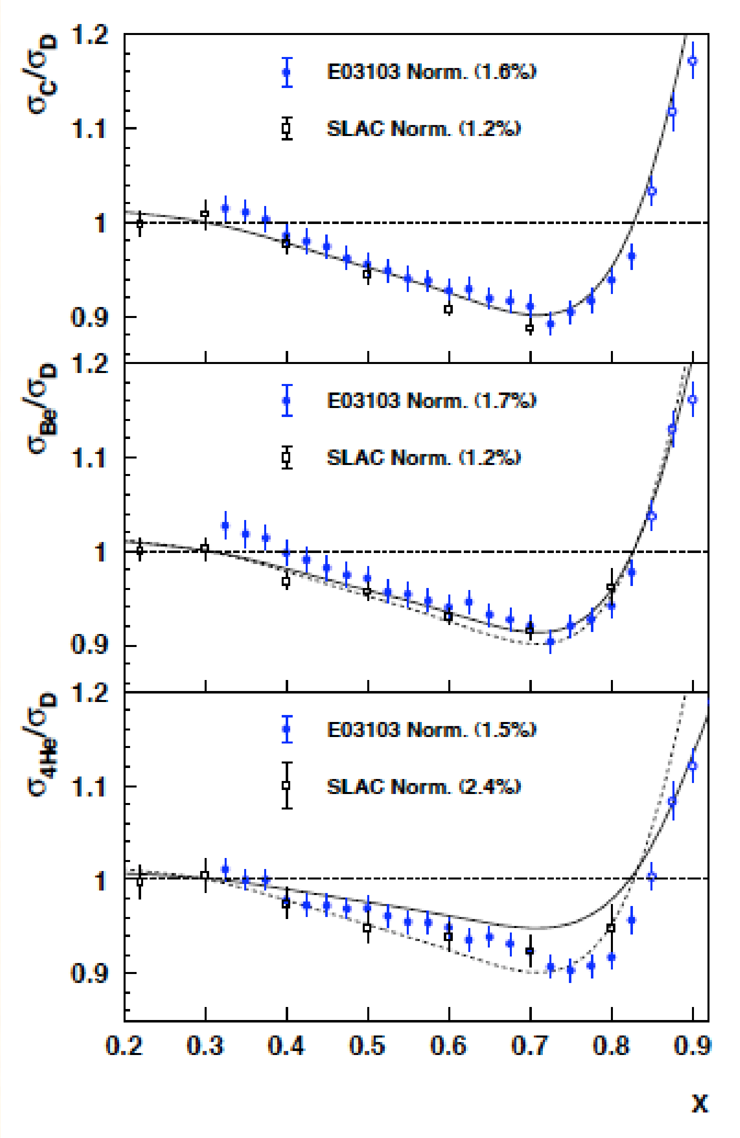
Future: MINERνA (He, C, Fe, Pb), …



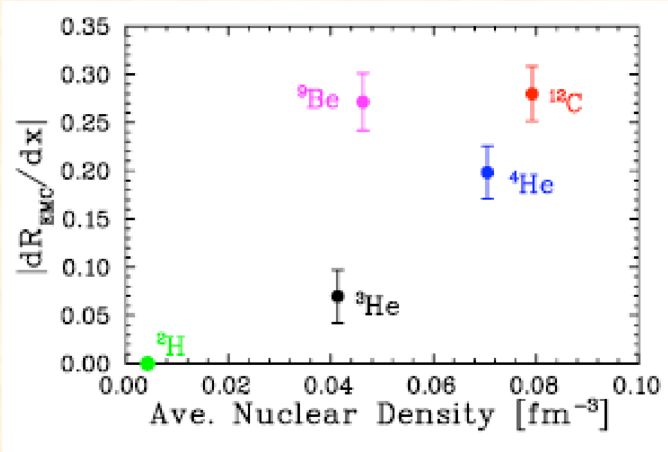


Recent measurements at JLab

J. Seely *et al.*,
Phys. Rev. Lett. 103 (2009) 202301.



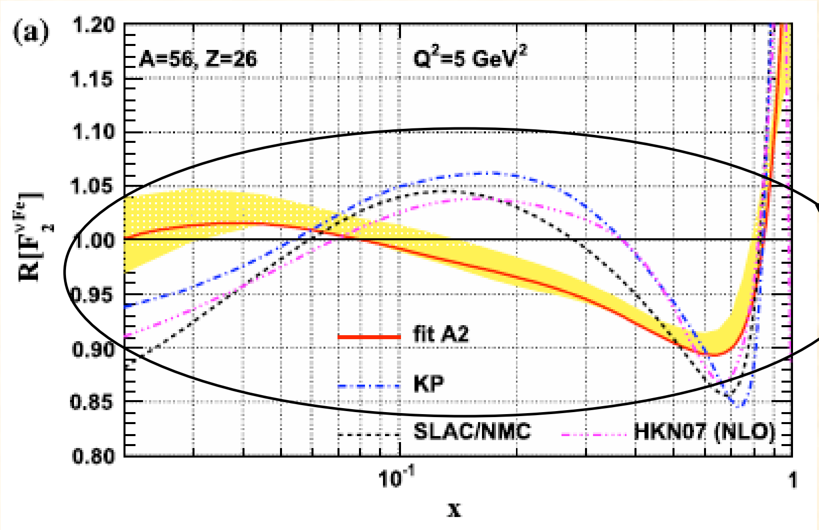
Results indicate that nuclear modifications may not be described by usual *A* (and density) dependence for light nuclei.



Issue of a modification difference between changed-lepton and neutrino reactions

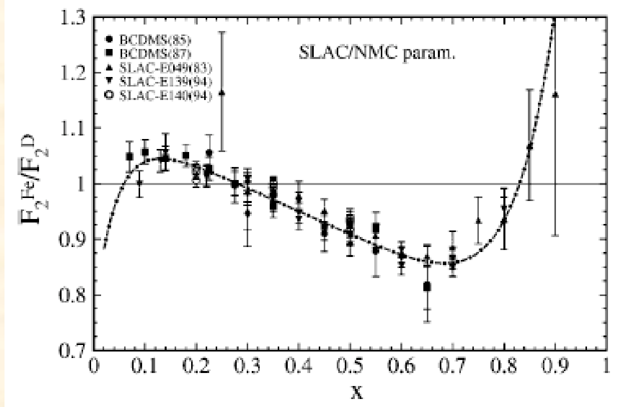
Analysis of SYKMOO-08 (Schienbein *et al.*)

SYKMOO-08 (I. Schienbein *et al.*),
PRD 77 (2008) 054013

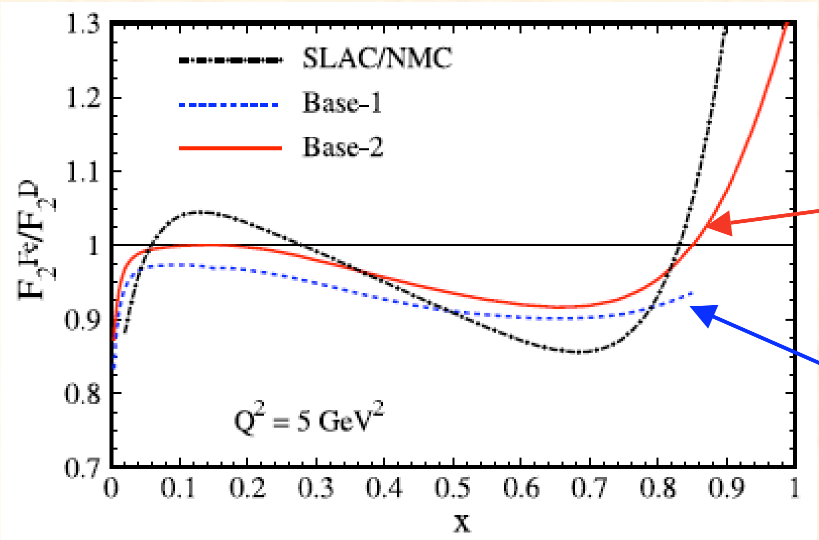


Differences from typical NPDFs.

Charged-lepton scattering



Neutrino scattering



Base-1
- remove CCFR data
- incorporate deuteron corrections

Base-2 corresponds to CTEQ6.1M with s≠sbar
- include CCFR data
  Charged-lepton correction factors are applied.
- s≠sbar

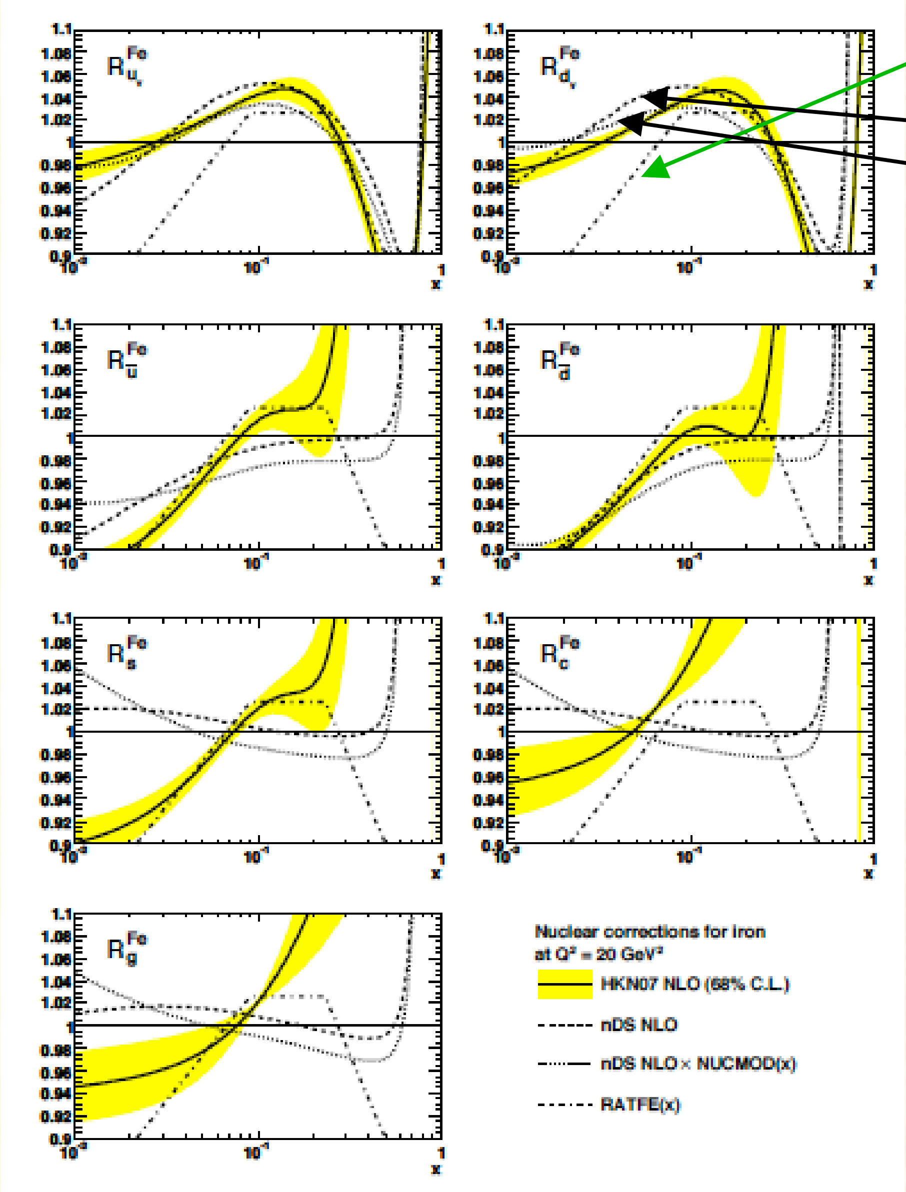
Base-2: Using current nucleonic PDFs, they (and MRST) obtained very different corrections from charged-lepton data.

Base-1: However, it depends on the analysis method for determining "nucleonic" PDFs.

MRSTW-08 analysis

HKN07

D. Martin, W. J. Stirling, R. S. Thorne, G. Watt, Eur. Phys. J. C 63, 189 (2009).



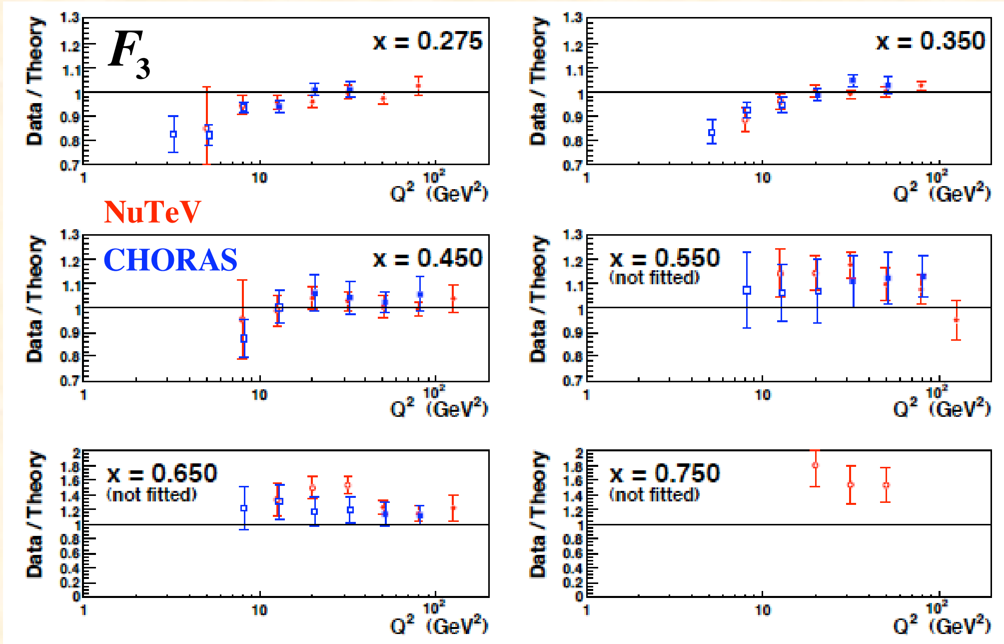
MRST98 nuclear correction

DS04

DS04*NUCMOD(x)

NUCMOD(x)

$$= \begin{cases} (1+0.03r_1)[1+0.015r_2 \ln^2(x_m/x)] & \text{for } x < x_m = e^{-2.5} \\ (1+0.03r_1)[1+0.015r_3 \ln^2(x/x_m)] & \text{for } x < x_m \end{cases}$$



Deviations at large x:

Same tendency as the Schienbein *et al.*'s.

Comments on related topics

- Nuclear modification effects on NuTeV $\sin^2\theta_W$ anomaly
- JLab ^{9}Be "anomaly" as a nuclear clustering aspect
- Analysis on tensor-polarized PDFs in the deuteron

Effects on NuTeV $\sin^2\theta_W$ anomaly due to nuclear modification differences between u_v and d_v

(1) S. Kumano, Phys. Rev. D66 (2002) 111301.

(2) M. Hirai, S. Kumano, T.-H. Nagai, Phys. Rev. D71 (2005) 113007.

Global analysis of F_2 and Drell-Yan data for $\varepsilon_v(x)$

$$u_v^A(x) = w_{u_v}(x,A)\,\frac{Z\,u_v(x) + N\,d_v(x)}{A}$$

$$d_v^A(x) = w_{d_v}(x,A)\,\frac{Z\,d_v(x) + N\,u_v(x)}{A}$$

$$\bar{q}^A(x) = w_{\bar{q}}(x,A)\,\bar{q}(x), \quad g^A(x) = w_g(x,A)\,g(x)$$

in the NPDF analysis

$$w_{uv} = 1 + (1-1/A^{1/3})\,\frac{a_{uv}+b_v x+c_v x^2+d_v x^3}{(1-x)^{\beta_v}}$$

$$w_{dv} = 1 + (1-1/A^{1/3})\,\frac{a_{dv}+b_v x+c_v x^2+d_v x^3}{(1-x)^{\beta_v}}$$

in the current analysis

$$w_{uv} + w_{dv} = 1 + (1-1/A^{1/3})\,\frac{a_v+b_v x+c_v x^2+d_v x^3}{(1-x)^{\beta_v}}$$

$$w_{uv} - w_{dv} = 1 + (1-1/A^{1/3})\,\frac{a_v'+b_v' x+c_v' x^2+d_v' x^3}{(1-x)^{\beta_v}}$$

Analysis result for $\varepsilon_v(x)$

$$\varepsilon_v(x) = \frac{w_{d_v}(x) - w_{u_v}(x)}{w_{d_v}(x) + w_{u_v}(x)}$$

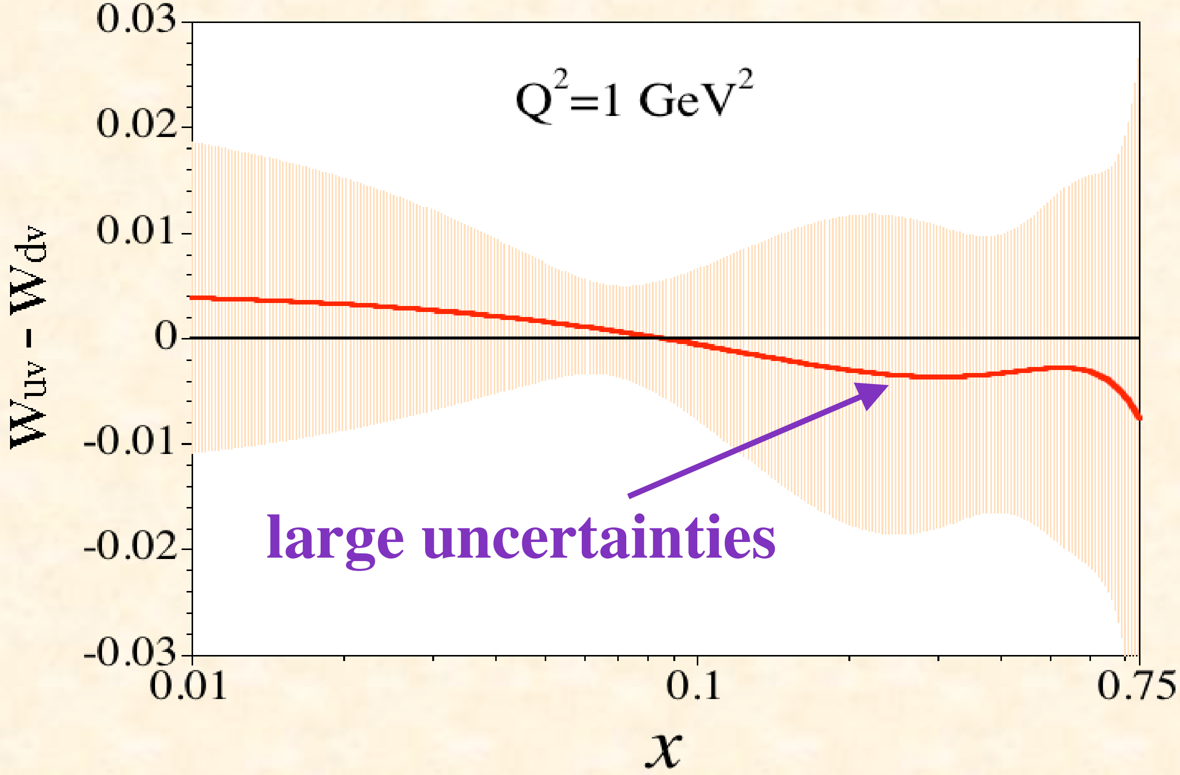
$$R_{\bar{A}} = \frac{1}{2} - \sin^2\theta_W - \varepsilon_v(x) \left\{ \left(\frac{1}{2} - \sin^2\theta_W\right) \frac{1+(1-y)^2}{1-(1-y)^2} - \frac{1}{3}\sin^2\theta_W \right\} + O(\varepsilon_v^2)$$

$$w_{uv} - w_{dv} = 1 + (1-1/A^{1/3}) \frac{a_v' + b_v' x + c_v' x^2 + d_v' x^3}{(1-x)^{\beta_v}}$$

a_v', b_v', c_v', d_v' are determined by the analysis

M. Hirai, SK, T.-H. Nagai, Phys. Rev. D71 (2005) 113007.

It is very difficult to determine the difference between nuclear modifications of u_v and d_v distributions at this stage.



Summary on NuTeV $\sin^2\theta_W$

(1) χ^2 analysis for the difference between nuclear modifications of u_v and d_v distributions.

It is very difficult to determine it at this stage.

(2) Effect on NuTeV $\sin^2\theta_W$

$$\Delta(\sin^2\theta_W) = 0.0004 \pm 0.0015 \qquad \text{(with a large error)}$$

JLab anomaly on ^{9}Be

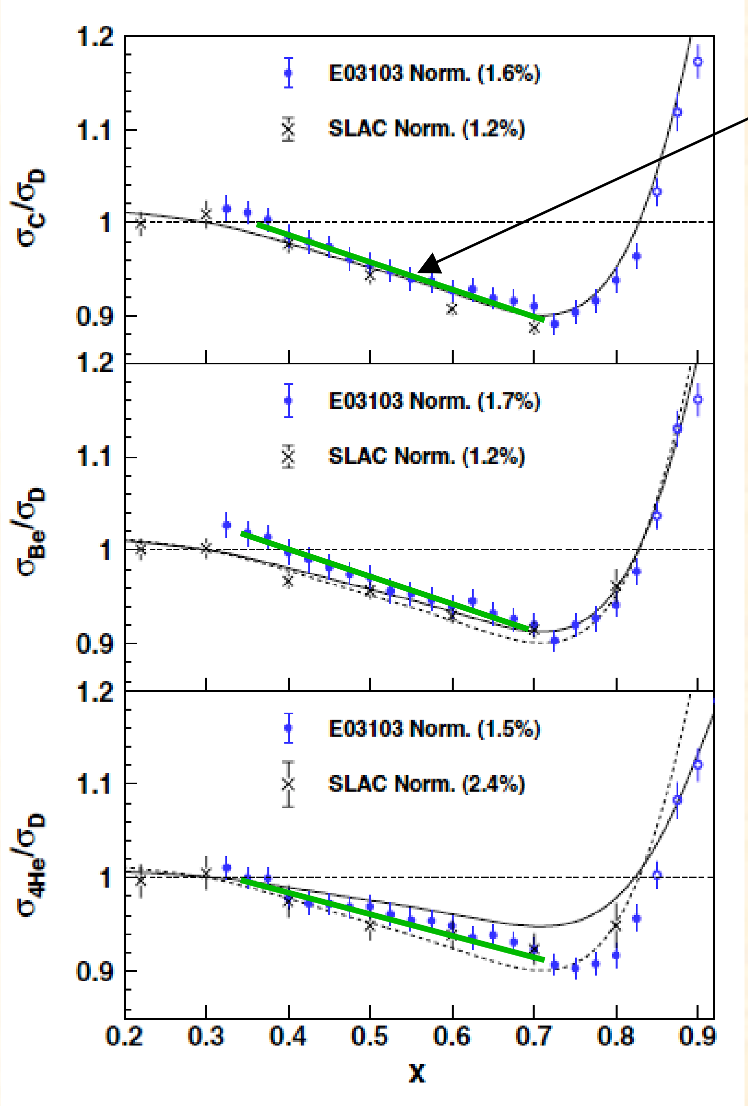
(A clustering aspect in DIS?)

M. Hirai, S. Kumano, K. Saito, and T. Watanabe

arXiv:1008.1313 [hep-ph]

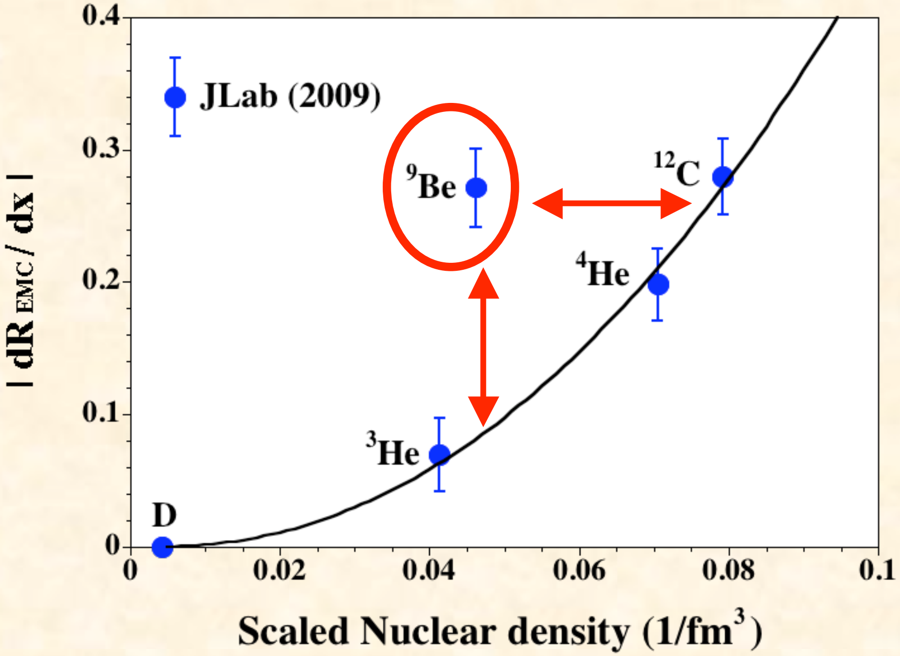
JLab "anomaly" on ^{9}Be

J. Seely *et al.*,
Phys. Rev. Lett. 103 (2009) 202301.



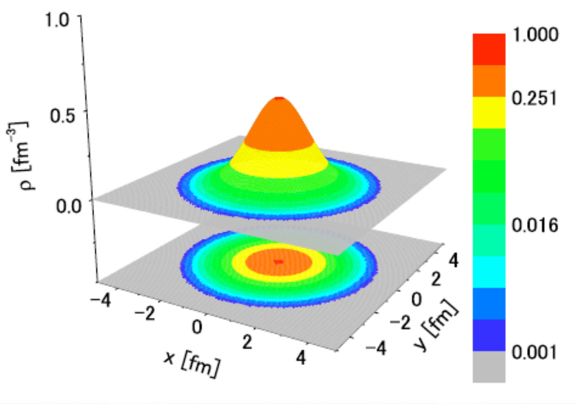
Slope: $\dfrac{dR_{EMC}}{dx}$, $R_{EMC} = \dfrac{\sigma_A}{\sigma_D}$

^{9}Be anomaly = EMC slope is too large
to be estimated from its nuclear density

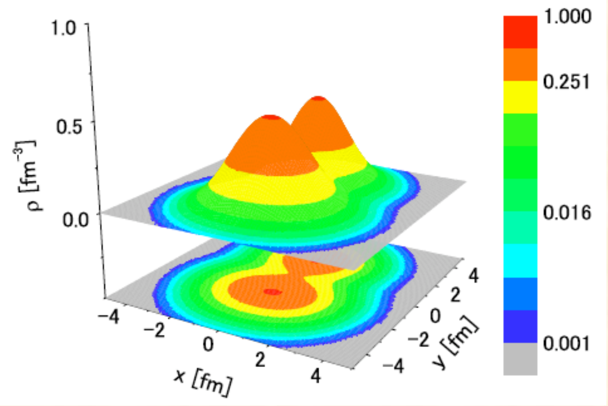


Cluster structure in ^{9}Be

Density distributions in ^{4}He and ^{9}Be



^{4}He



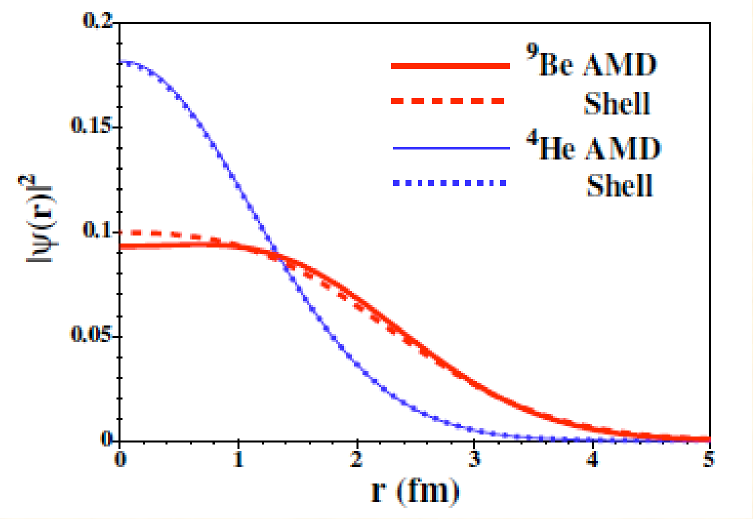
^{9}Be (~ ^{4}He + ^{4}He + n)

Two models:

(1) AMD (antisymmetrized molecular dynamics) to describe clustering structure

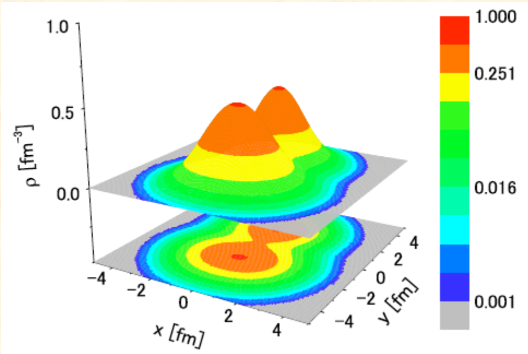
(2) Shell model

Space (r) distributions

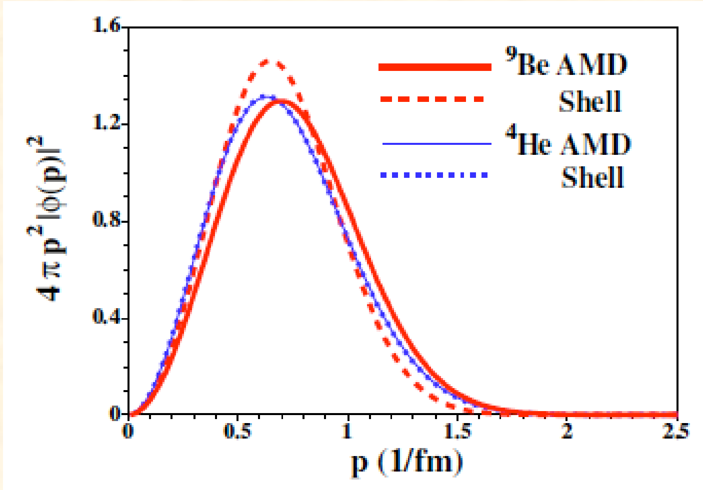


However, if the densities are averaged over the polar and azimuthal angles, differences from shell structure are not so obvious although there are some differences in ^{9}Be in comparison with ^{4}He.

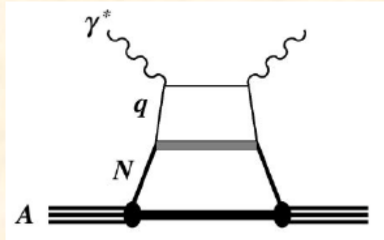
EMC effect



Momentum (p) distributions

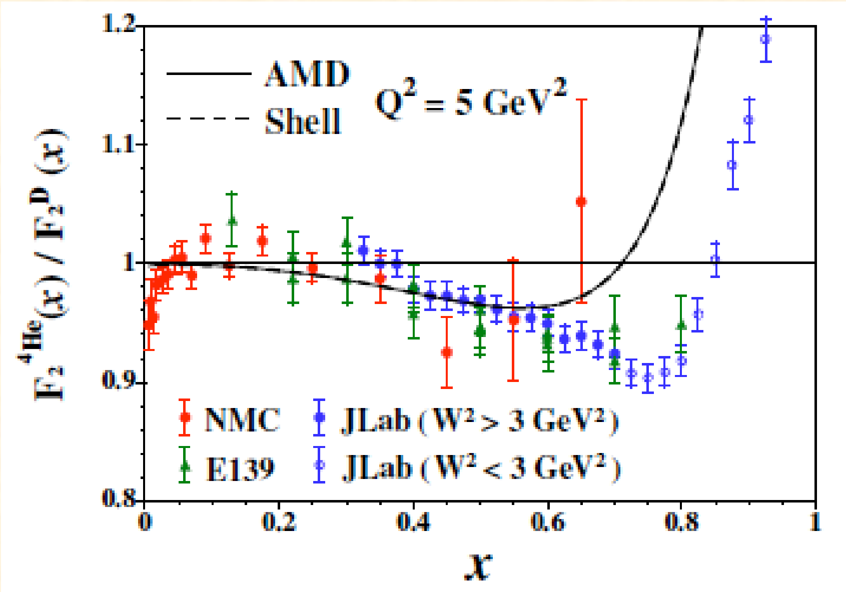


Simple convolution model

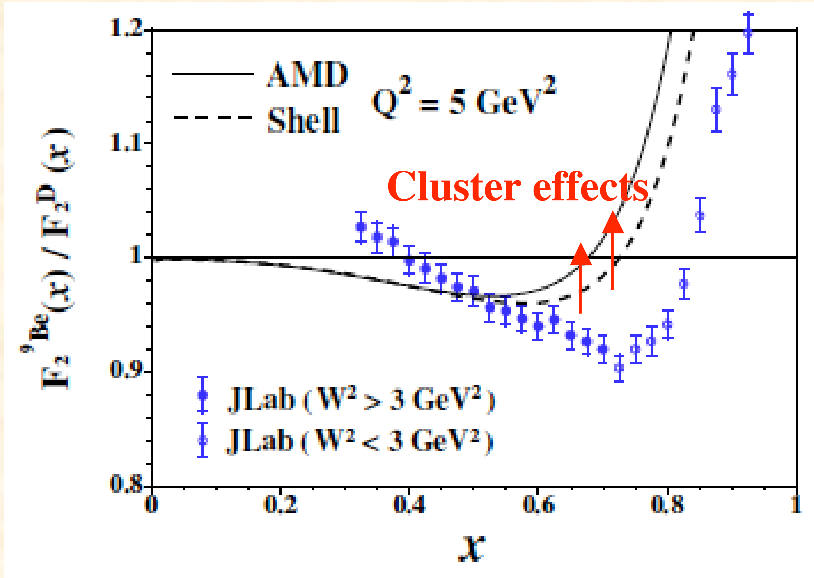


$$F_2^A(x, Q^2) = \int_x^A dy\, f(y)\, F_2^N(x/y, Q^2)$$

^{4}He

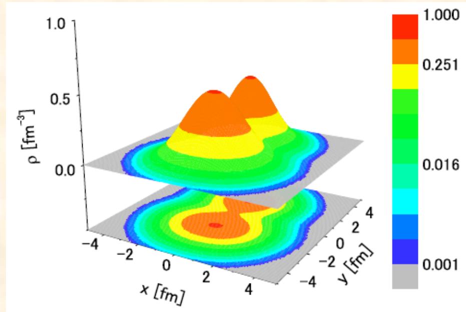


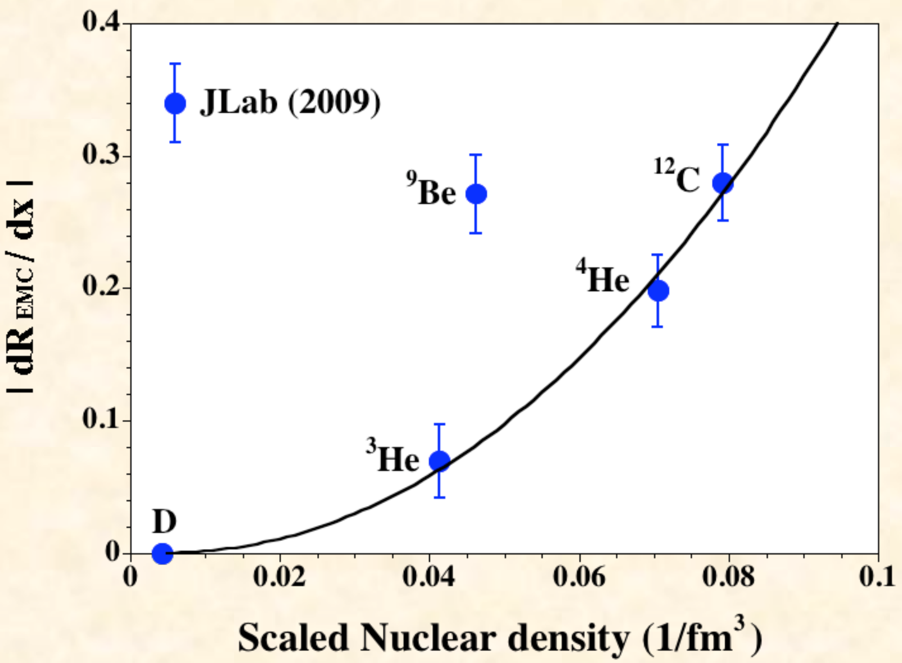
^{9}Be

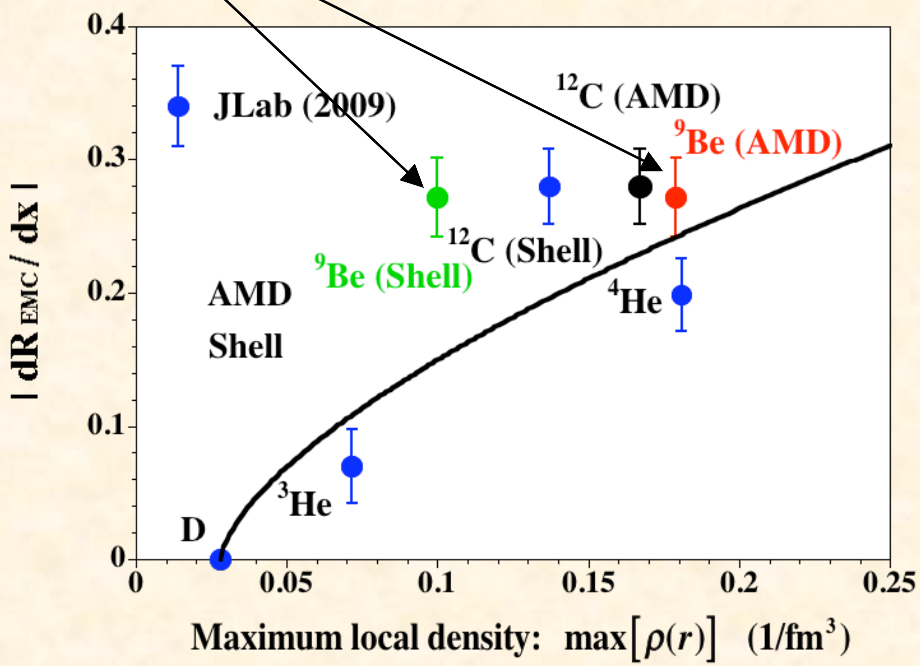


EMC slopes plotted by maximum local densities

The ^{9}Be anomaly can be explained
by the high-densities, which are created
by clustering in the ^{9}Be nucleus.







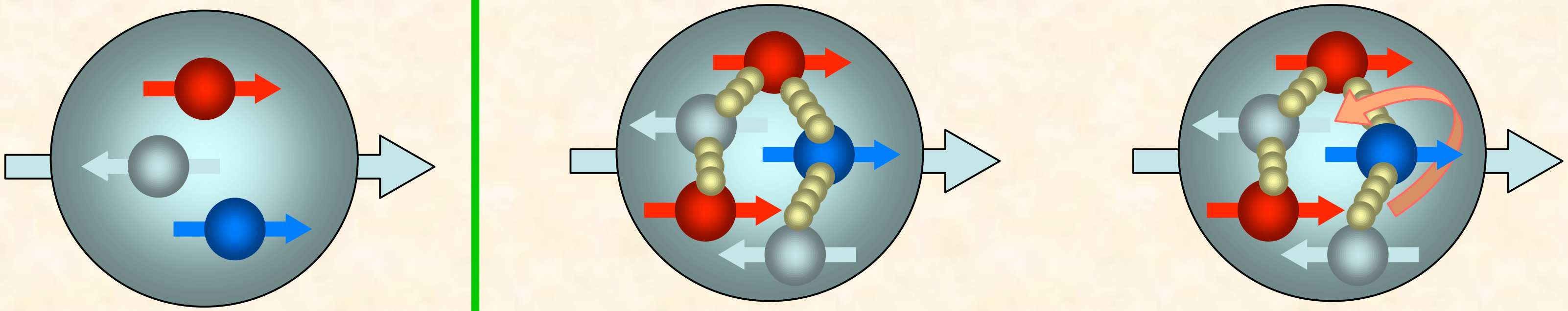
Original figure → **Plotted by the maximum local densities**

Tensor-polarized Parton Distribution Functions in the Deuteron

S. Kumano, Phys. Rev. D 82 (2010) 017501

Nucleon spin

Almost none of nucleon spin is carried by quarks! → **Nucleon spin crisis!?**

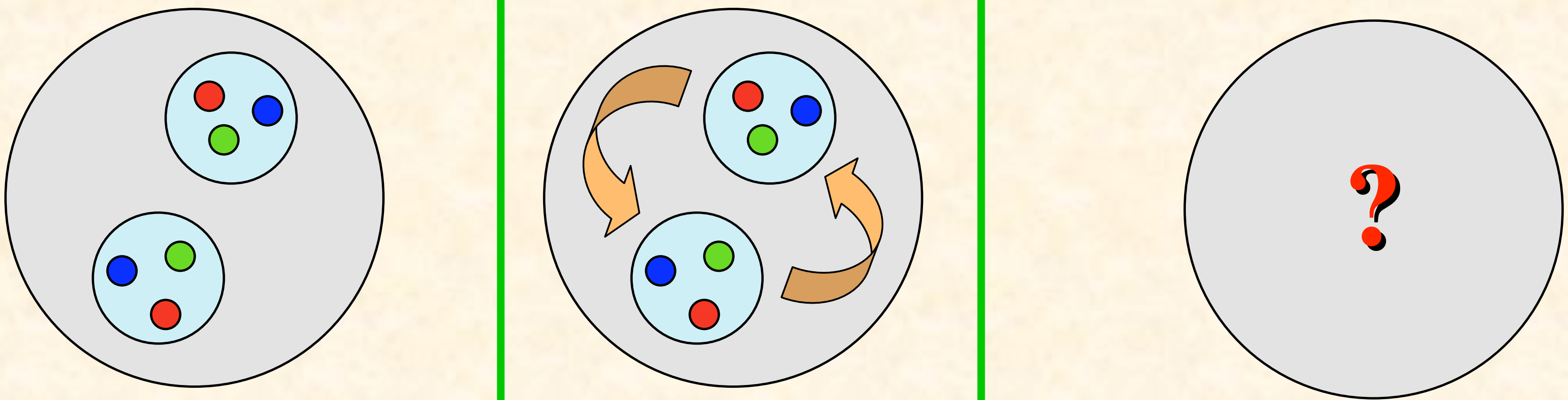


Naïve Quark Model | Sea-quarks and gluons? | Orbital angular momenta ?

"old" standard model

Tensor structure b_1 (*e.g.* deuteron)

Tensor-structure crisis!?



only S wave

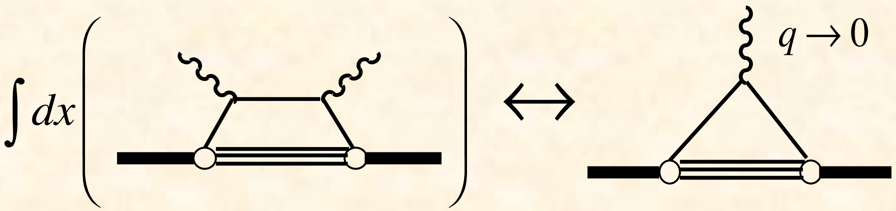
$b_1 = 0$

S + D waves

standard model $b_1 \neq 0$

$b_1^{\text{experiment}} \neq b_1^{\text{"standard model"}}$

Constraint on valence-tensor polarization (sum rule)



F.E.Close and SK, PRD42, 2377 (1990).

$$\int dx\, b_1^D(x) = \frac{5}{18}\int dx\left[\delta_T u_v + \delta_T d_v\right] + \frac{1}{18}\int dx\left[8\delta_T \bar{u}^D + 2\delta_T \bar{d}^D + \delta_T \bar{s}^D\right]$$

Elastic amplitude in a parton model

$$\Gamma_{H,H} = \langle p,H \,|\, J_0(0) \,|\, p,H\rangle = \sum_i e_i \int dx\left[q_{i\uparrow}^H + q_{i\downarrow}^H - \bar{q}_{i\Uparrow}^H - \bar{q}_{i\Downarrow}^H\right]$$

$$\frac{1}{2}\left[\Gamma_{0,0} - \frac{1}{2}\left(\Gamma_{1,1} + \Gamma_{-1,-1}\right)\right] = \frac{1}{3}\int dx\left[\delta_T u_v(x) + \delta_T d_v(x)\right]$$

Macroscopically $\Gamma_{0,0} = \lim_{t\to 0}\left[F_c(t) - \frac{t}{3}F_Q(t)\right],\quad \Gamma_{+1,+1} = \Gamma_{-1,-1} = \lim_{t\to 0}\left[F_c(t) + \frac{t}{6}F_Q(t)\right]$

$$\frac{1}{2}\left[\Gamma_{0,0} - \frac{1}{2}\left(\Gamma_{1,1} + \Gamma_{-1,-1}\right)\right] = -\lim_{t\to 0}\frac{t}{2}F_Q(t)$$

$$\int dx\, b_1^D(x) = \frac{5}{9}\frac{3}{2}\left[\Gamma_{0,0} - \frac{1}{2}\left(\Gamma_{1,1} + \Gamma_{-1,-1}\right)\right] + \frac{1}{18}\int dx\left[8\delta_T \bar{u}^D + 2\delta_T \bar{d}^D + \delta_T \bar{s}^D\right]$$

$$= -\frac{5}{6}\lim_{t\to 0} tF_Q(t) + \frac{1}{18}\int dx\left[8\delta_T \bar{u}^D + 2\delta_T \bar{d}^D + \delta_T \bar{s}^D\right]$$

$$= 0\ \text{(valence)} + \frac{1}{18}\int dx\left[8\delta_T \bar{u}^D + 2\delta_T \bar{d}^D + \delta_T \bar{s}^D\right]$$

Constraint on tensor-polarized valence quarks: $\int dx\, \delta_T q_v(x) = 0$

Functional form of parametrization

$$\delta_T q = q^0 - \frac{q^{+1} + q^{-1}}{2}$$

Assume flavor-symmetric antiqurk distributions: $\delta \bar{q}^D \equiv \delta \bar{u}^D = \delta \bar{d}^D = \delta s^D = \delta \bar{s}^D$

$$b_1^D(x)_{LO} = \frac{1}{18}\left[4\delta_T u_v^D(x) + \delta_T d_v^D(x) + 12\ \delta_T \bar{q}^D(x)\right]$$

At $Q_0^2 = 2.5\ \text{GeV}^2$, $\delta_T q_v^D(x,Q_0^2) = \delta_T w(x) q_v^D(x,Q_0^2)$, $\delta_T \bar{q}^D(x,Q_0^2) = \alpha_{\bar{q}} \delta_T w(x) \bar{q}^D(x,Q_0^2)$

Certain fractions of quark and antiquark distributions are tensor polarized and such probabilities are given by the function $\delta_T w(x)$ **and an additional constant** $\alpha_{\bar{q}}$ **for antiquarks in comparison with the quark polarization.**

$$b_1^D(x,Q_0^2)_{LO} = \frac{1}{18}\left[4\delta_T u_v^D(x,Q_0^2) + \delta_T d_v^D(x,Q_0^2) + 12\delta_T \bar{q}^D(x,Q_0^2)\right]$$

$$= \frac{1}{36}\delta_T w(x)\left[5\left\{u_v(x,Q_0^2) + d_v(x,Q_0^2)\right\} + 4a_{\bar{q}}\left\{2\bar{u}(x,Q_0^2) + 2\bar{d}(x,Q_0^2) + s(x,Q_0^2) + \bar{s}(x,Q_0^2)\right\}\right]$$

$$\delta_T w(x) = a x^b (1-x)^c (x_0 - x)$$

Two types of analyses

Set 1: $\delta_T \bar{q}^D(x) = 0$ **Tensor-polarized antiquark distributions are terminated** $(\alpha_{\bar{q}} = 0)$,

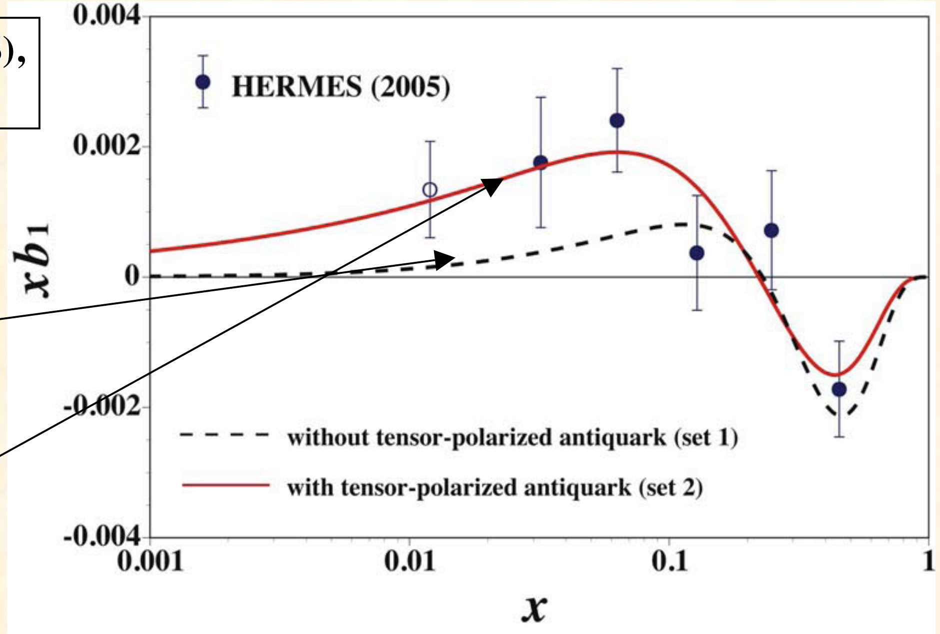
Set 2: $\delta_T \bar{q}^D(x) \neq 0$ **Finite tensor-polarized antiquark distributions are allowed** $(\alpha_{\bar{q}} \neq 0)$.

Results

Data: A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.

Two-types of fit results:

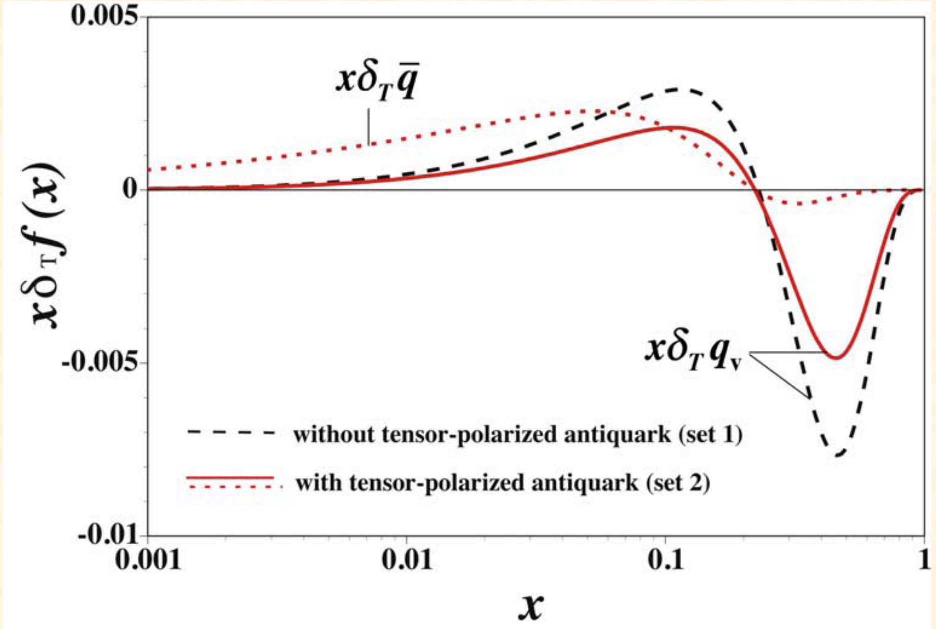
- **set-1:** χ^2 / d.o.f. = 2.83
  Without $\delta_T q$, the fit is not good enough.
- **set-2:** χ^2 / d.o.f. = 1.57
  With finite $\delta_T q$, the fit is reasonably good.



Obtained tensor-polarized distributions
$\delta_T q(x)$, $\delta_T \bar{q}(x)$ from the HERMES data.

→ They could be used for

- experimental proposals,
- comparison with theoretical models.



Finite tensor polarization for antiquarks:

$$\int_0^1 dx b_1(x) = 0.058$$
$$= \frac{1}{9}\int_0^1 dx\left[4\delta_T \bar{u}(x) + \delta_T \bar{d}(x) + \delta_T \bar{s}(x)\right]$$

Summary

(1) The tensor-polarized distributions: $\delta_T q(x),\ \delta_T \bar{q}(x)$
were obtained from the HERMES data on b_1.

(2) Finite tensor polarization was obtained
for antiquarks: $\int dx \delta_T \bar{q}(x) \neq 0$.

Prospects

Future experimental possibilities at JLab, J-PARC, RHIC, COMPASS, GSI-FAIR, …

Unpolarized proton+ polarized deuteron

Spin asymmetry in $p + \vec{d} \to \mu^+ \mu^- + X$

$$A_{UQ_0} = \frac{\sum_a e_a^2 \left[q_a(x_A) \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta_T q_a(x_B) \right]}{\sum_a e_a^2 \left[q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]}$$

Polarized proton-deuteron Drell-Yan (Theory) S. Hino and SK, PR D 59 (1999) 094026, D 60 (1999) 054018.

Unique advantage of J-PARC ($\delta \bar{q}$ measurement)

$$\int dx\, b_1^D(x) = 0 + \frac{1}{9} \int dx\, \delta_T \bar{q}(x)$$

$$A_{UQ_0}(\text{large } x_F) \approx \frac{\sum_a e_a^2 q_a(x_A) \delta \bar{q}_a(x_B)}{\sum_a e_a^2 q_a(x_A) \bar{q}_a(x_B)}$$

Gottfried: $\int \frac{dx}{x} \left[F_2^p(x) - F_2^n(x) \right] = \frac{1}{3} + \frac{2}{3} \int dx \left[\bar{u} - \bar{d} \right]$

The End

The End